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How do fundamental modelling choices affect radial diffusion in Earth's Radiation Belts?

S. N. Bentley[1,2], J. R. Stout[2], R. L. Thompson[2], D. J. Ratliff[1], C. E. J. Watt[1,2]

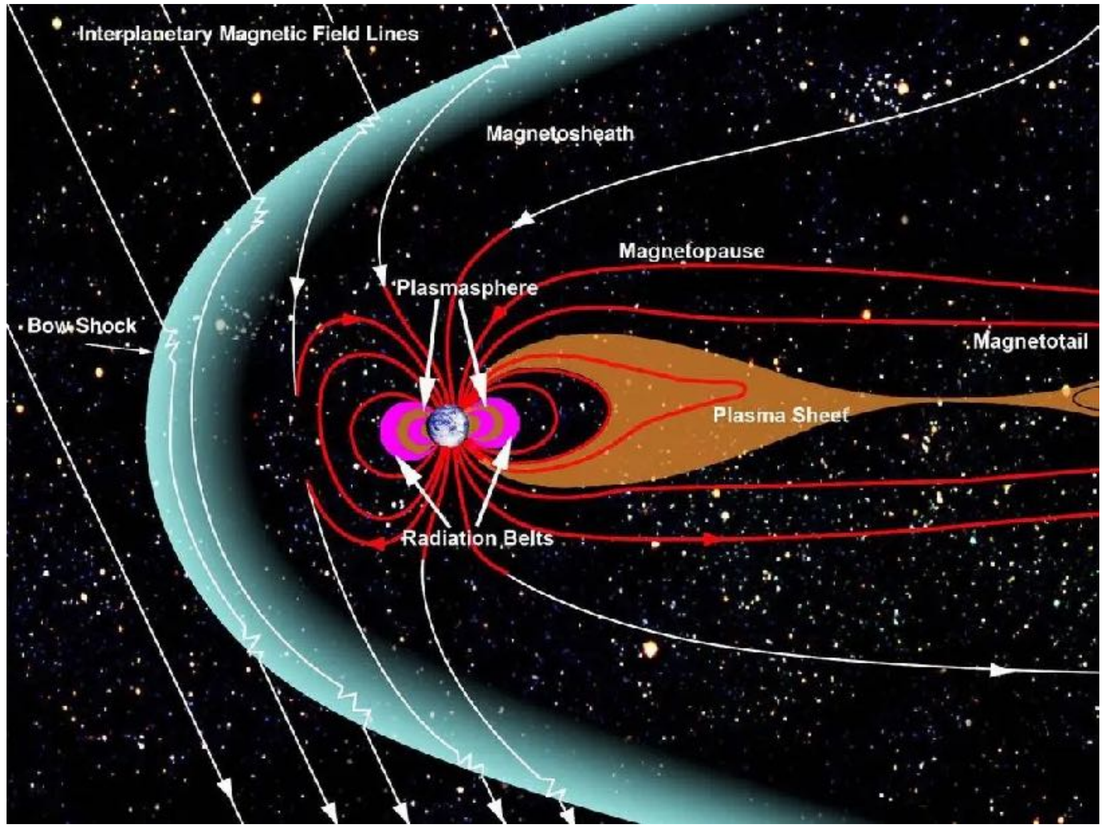
[1]Northumbria University, [2]University of Reading.

Bentley et al. 2024, Physics of Plasmas; https://doi.org/10.1063/5.0217268

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What's this all about?

- In radiation belt modelling, we need ensemble modelling to capture uncertainty and chaotic behaviour.
- **BUT** we don't even know how changing basic diffusion model settings (outer edges, starting values of particles density etc.) changes the final distribution!
- **THEREFORE** we need to run these simulations to compare them, and develop a way to compare them.



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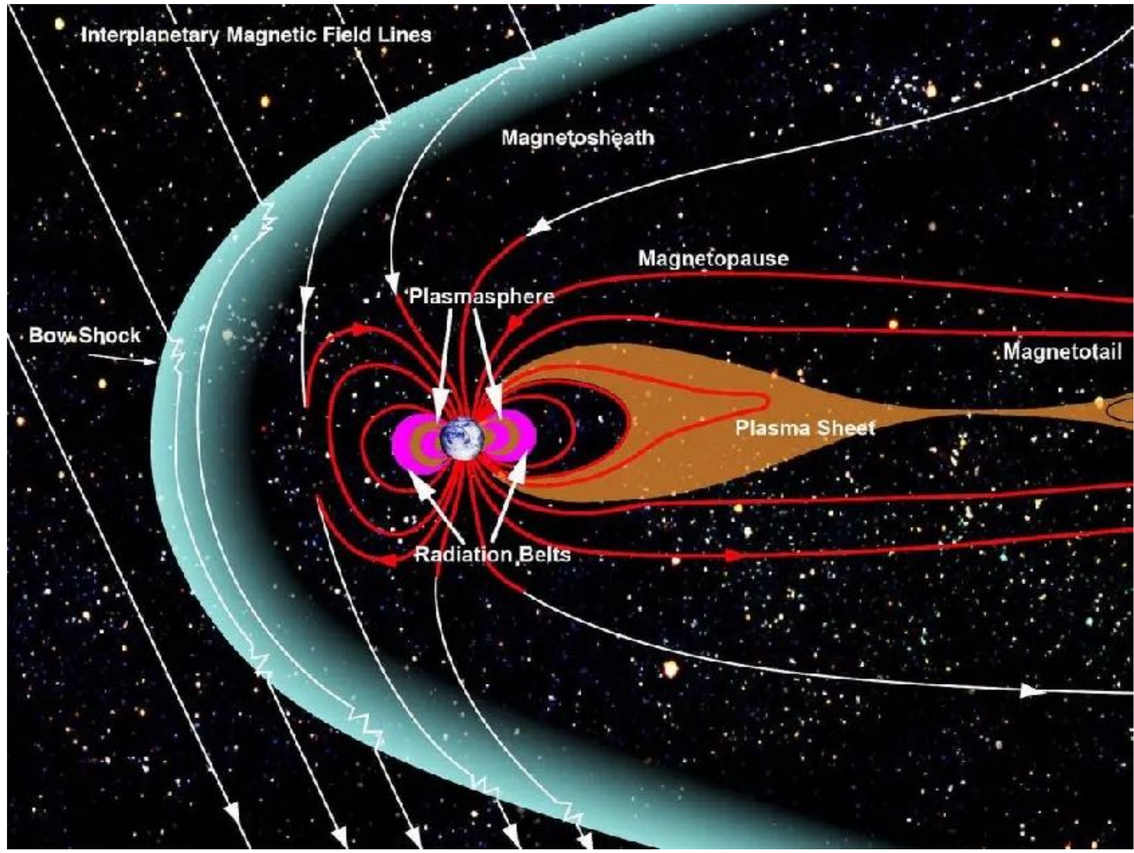
The near-Earth plasma environment

Solar wind → Earth's magnetosphere → radiation belts → space weather

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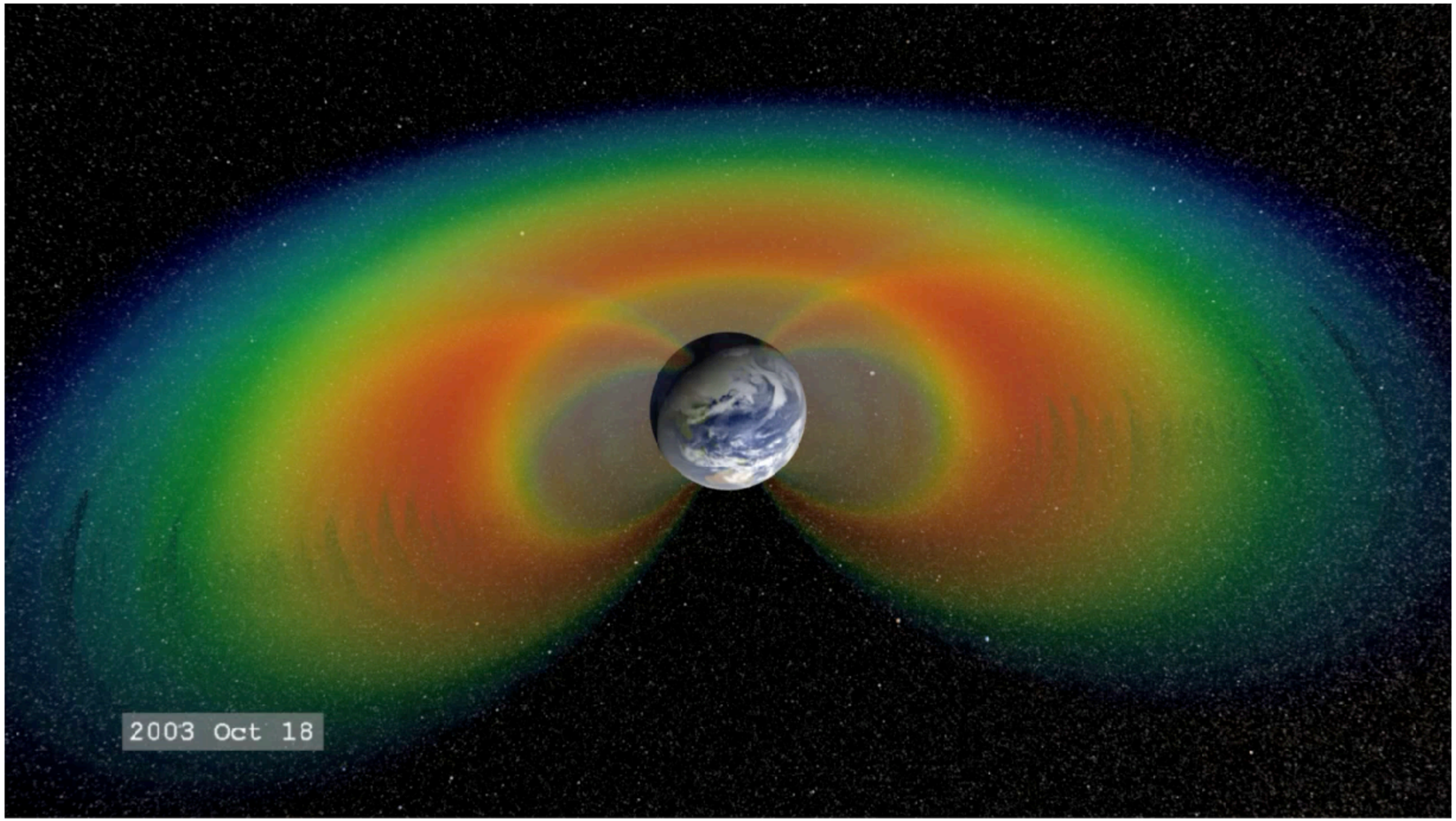
Near-Earth space as a plasma laboratory

- Solar wind: predominantly electrons, protons and α -particles, 200-1000 km s^{-1}.
- The magnetosphere: lots of different particle populations at various energies.
- Mostly collisionless.



Earth’s radiation belts

- Particles trapped by the Earth’s magnetic field
- Inner belt is mostly protons 1-3 R_E, fairly stable
- Outer belt is highly variable, electrons, 3-10 R_E
- keV - MeV

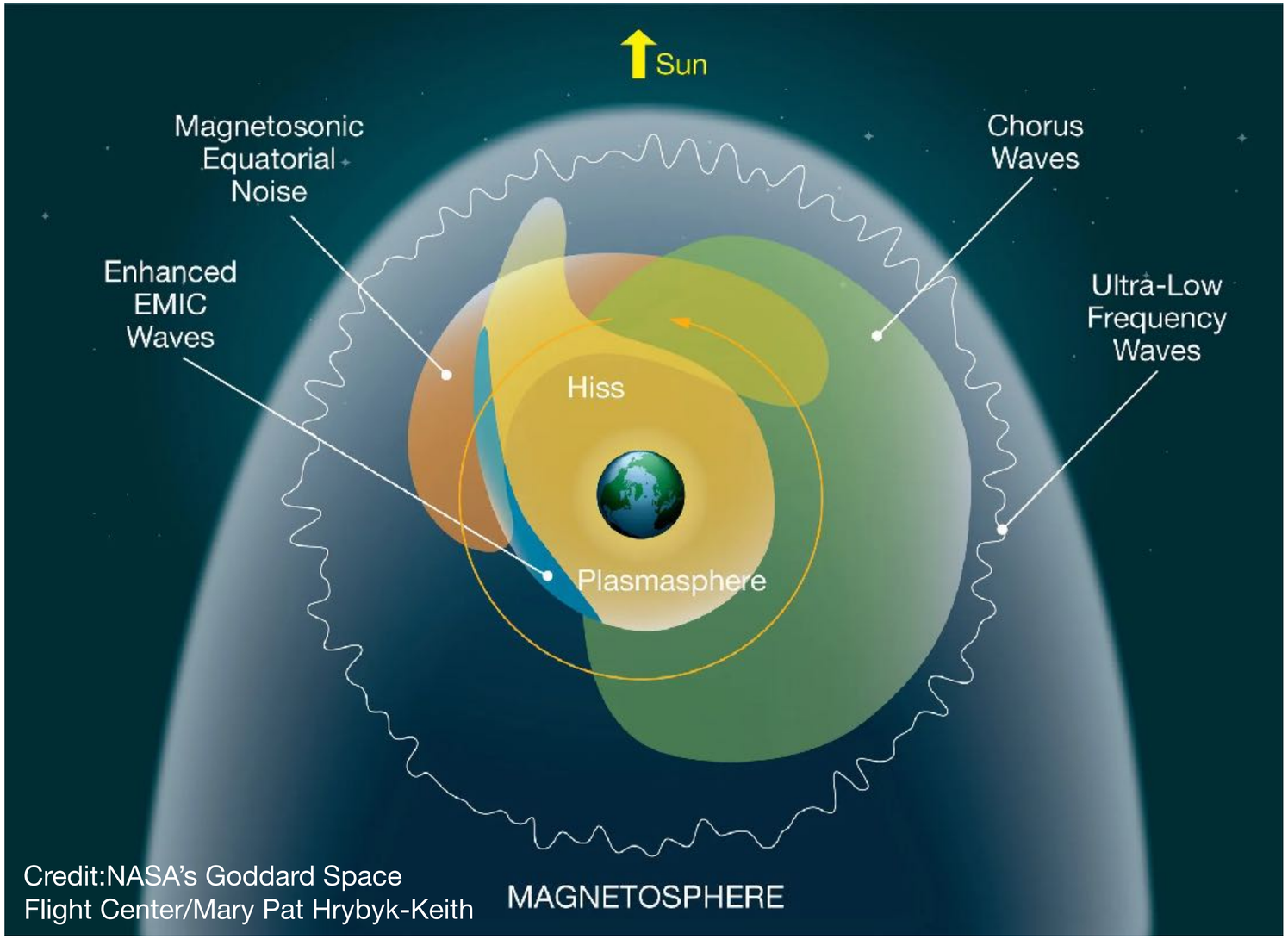


Earth's radiation belts

…are interesting:

#1: Wave-particle interactions

- So few particles that energy cannot be transferred through collisions
- Instead: specific wave-particle interactions



Credit:NASA's Goddard Space Flight Center/Mary Pat Hrybyk-Keith

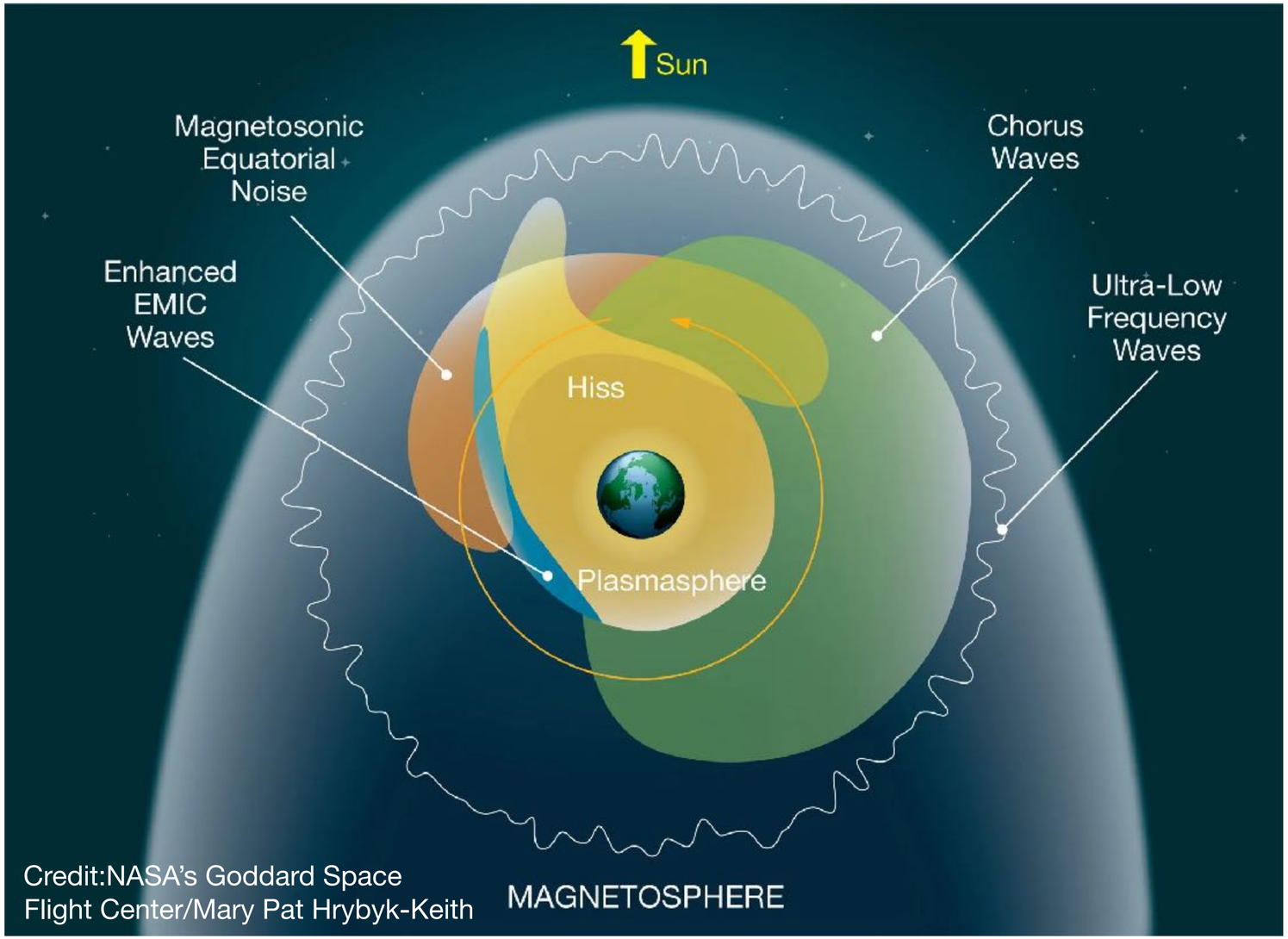
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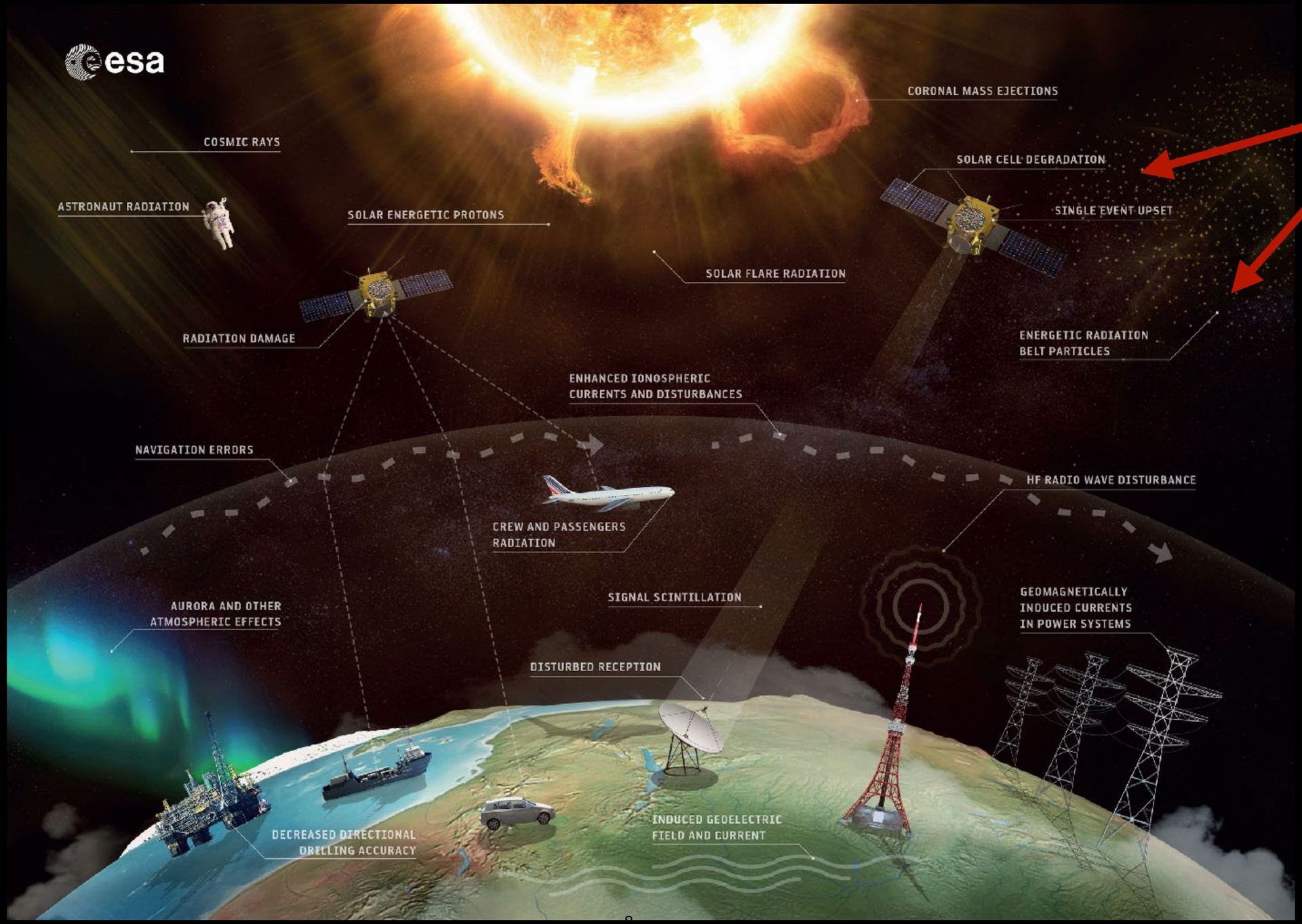
#1: Wave-particle interactions

- So few particles that energy cannot be transferred through collisions
- Instead: specific wave-particle interactions

#2: Space weather



Credit:NASA's Goddard Space Flight Center/Mary Pat Hrybyk-Keith


esa
CORONAL MASS EJECTIONS
COSMIC RAYS
SOLAR CELL DEGRADATION
ASTRONAUT RADIATION
SINGLE EVENT UPSET
SOLAR ENERGETIC PROTONS
SOLAR FLARE RADIATION
RADIATION DAMAGE
ENERGETIC RADIATION
BELT PARTICLES
ENHANCED IONOSPHERIC
CURRENTS AND DISTURBANCES
NAVIGATION ERRORS
HF RADIO WAVE DISTURBANCE
CREW AND PASSENGERS
RADIATION
AURORA AND OTHER
ATMOSPHERIC EFFECTS
SIGNAL SCINTILLATION
GEOMAGNETICALLY
INDUCED CURRENTS
IN POWER SYSTEMS
DISTURBED RECEPTION
INDUCED GEOELECTRIC
FIELD AND CURRENT
DECREASED DIRECTIONAL
DRILLING ACCURACY

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How can we model the radiation belts?

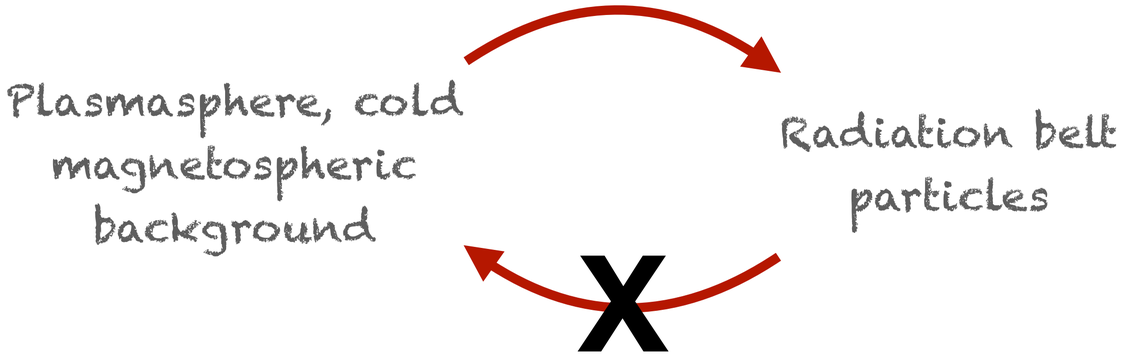
Particle motions → diffusion equation → radial diffusion

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How can we model the radiation belts?

(and what is radial diffusion??)

- Very high energy, but few particles: We can treat the radiation belts as if they “sit on top” of the rest of the magnetosphere
- **Kinetic modelling** of each particle is possible but expensive.
- Instead: Use convenient facts about the high energy particles to model the radiation belt population via the very efficient **diffusion modelling**.



$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x_1^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2}\right)$$

$$\frac{\partial u}{\partial t} = D\nabla u$$

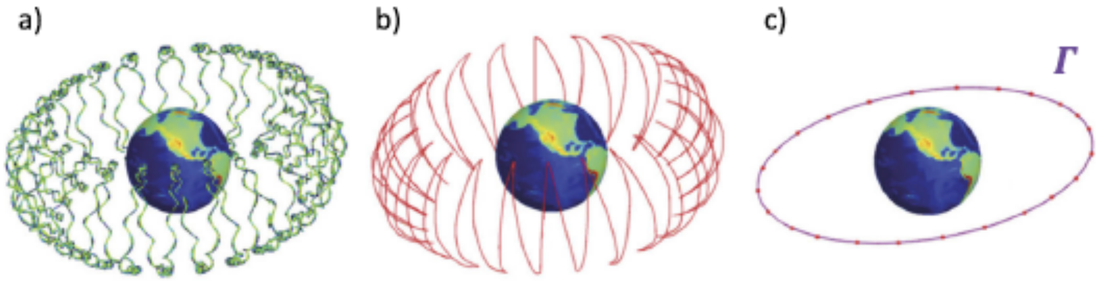
The diffusion (or heat) equation (here, with constant diffusion coefficient D) is very well known and can be used instead.

Three Particle Motions

- Classical physics: If you are moving adiabatically on a periodic path, there is a conserved quantity J associated with that motion.

adiabatic: slowly changing, in a reversible manner

- If more interesting things happen (i.e. on the timescale of that motion, or faster) the J_i of that motion will change



Three particle motions (Lejosne). So there are three conserved quantities, each one associates with the timescale of one periodic motion.

	Motion	~Timescale
J_1 = MU	Gyromotion	<0.01s
J_2	Bounce	<1s
J_3 = PHI ~ L*	Drift	>10s

General radiation belt diffusion equation

$$\frac{\partial f}{\partial t} = \sum_{ij} \frac{\partial}{\partial J_i}\left[D_{ij}\frac{\partial f}{\partial J_j}\right] \qquad D_{ij} \sim \begin{pmatrix} D_{\alpha\alpha} & D_{\alpha E} & 0 \\ D_{E\alpha} & D_{EE} & 0 \\ 0 & 0 & D_{LL} \end{pmatrix}$$

- Let's expand the diffusion equation
- The full radiation belt quasilinear diffusion equation uses all three invariants, where we assume that radial diffusion does not interact with energy or pitch angle diffusion

$$\frac{\partial f}{\partial t} = \frac{1}{g(\alpha)}\frac{\partial}{\partial \alpha}\bigg|_{E,L} g(\alpha)\left(D_{\alpha\alpha}\frac{\partial}{\partial \alpha}\bigg|_{E,L} + D_{EE}\frac{\partial}{\partial E}\bigg|_{\alpha,L}\right) + \frac{1}{A(E)}\frac{\partial}{\partial E}\bigg|_{\alpha,L} A(E)\left(D_{EE}\frac{\partial}{\partial E}\bigg|_{\alpha,L} + D_{E\alpha}\frac{\partial}{\partial \alpha}\bigg|_{E,L}\right) + L^2\frac{\partial}{\partial L}\bigg|_{\mu,J}\left(\frac{D_{LL}}{L^2}\frac{\partial f}{\partial L}\bigg|_{\mu,J}\right)$$

- But if we only care about longer timescales, we end up with the 1D radial diffusion equation

Schulz & Lanzerotti 1974, Walt 2009, Glauert et al. 2014

General radiation belt diffusion equation

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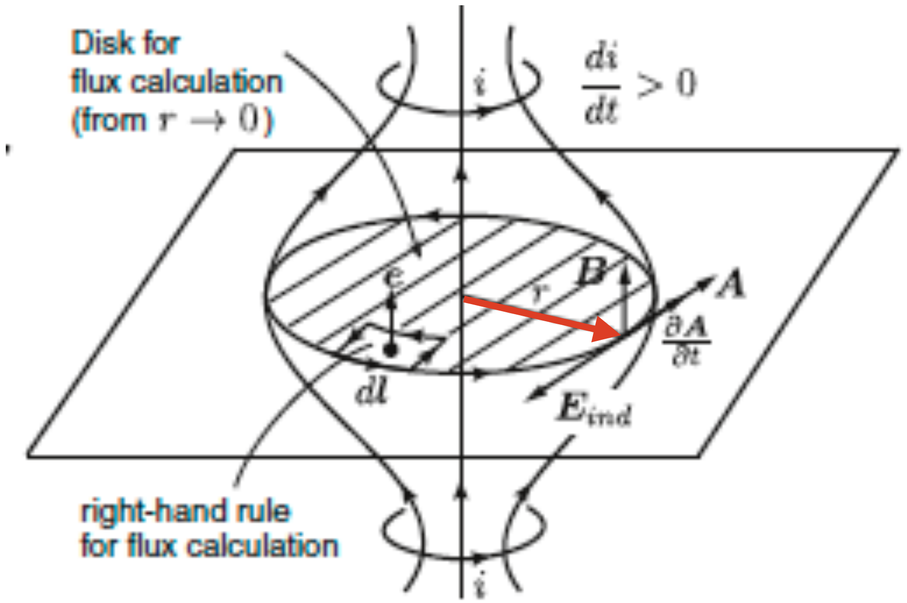
Schulz & Lanzerotti 1974, Walt 2009, Glauert et al. 2014

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Example: Radial diffusion

1. The slowest motion is drift around the Earth (~minutes, hours)The conserved quantity relating to this is the **magnetic flux contained in a drift orbit.**

2. You can also label this by the radius of that drift orbit, if the magnetic field were relaxed to a dipole, *L*.* **The drift shell L* can be used as the third adiabatic invariant.**



The adiabatic invariant Φ or L*. (Roederer & Zhang 2014) Under slow changes to E, B, the magnetic flux contained a drift shell is conserved. You can label this using L* (roughly *r* in the diagram above)

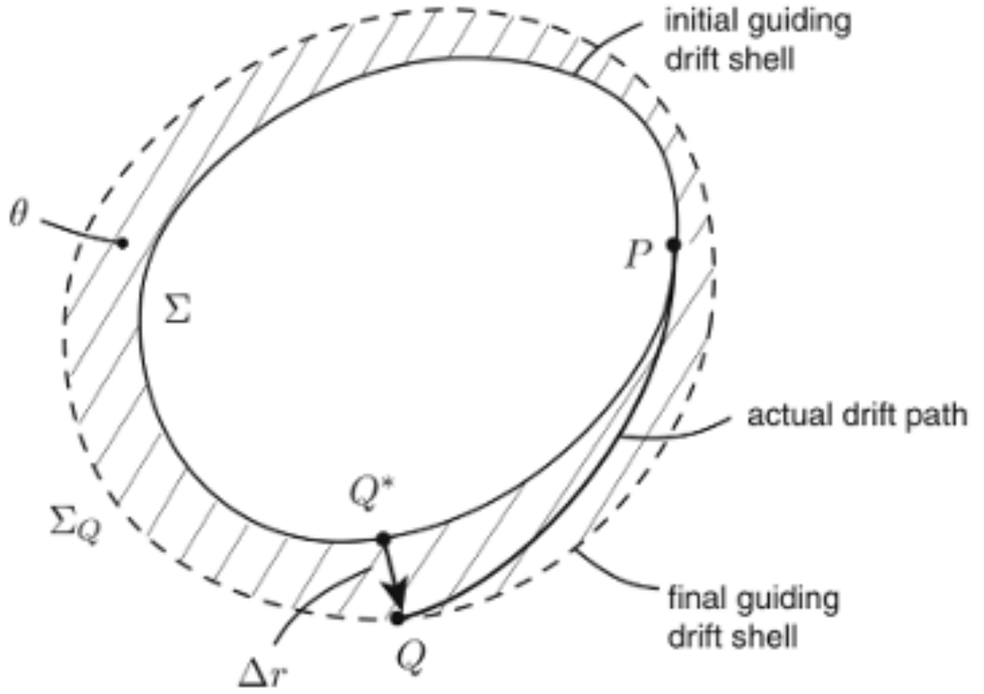
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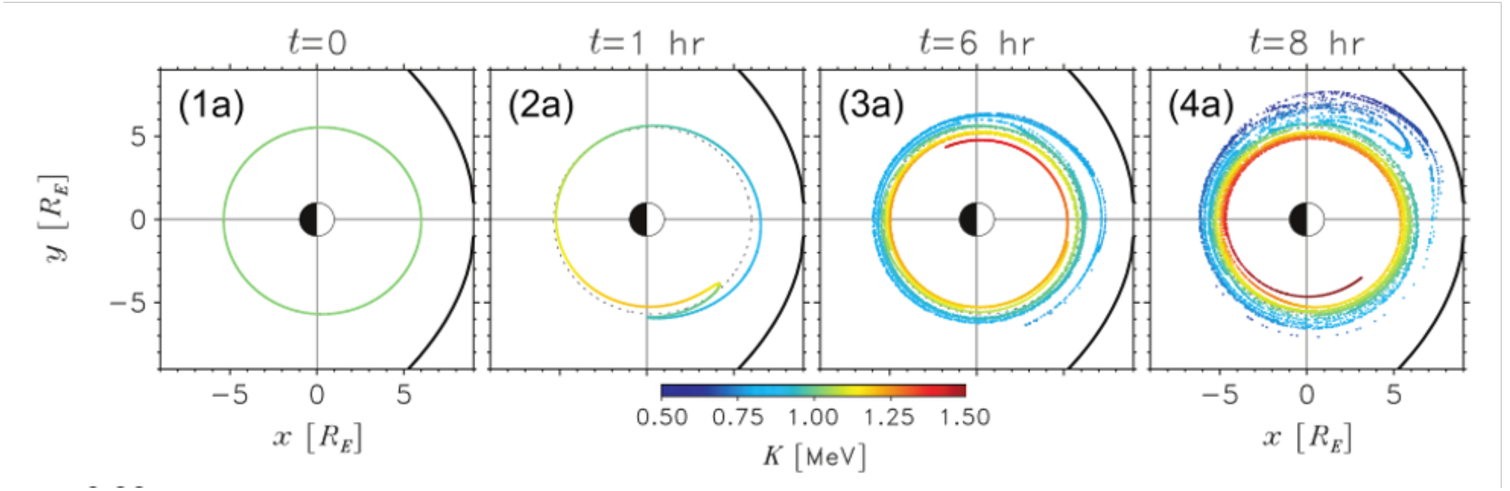
3. If an individual particle gets kicked off their drift orbit by E, B perturbations, then their *L** changes.

4. The whole population gets kicked to nearby orbits -> show distribution across *L** using a diffusion equation.



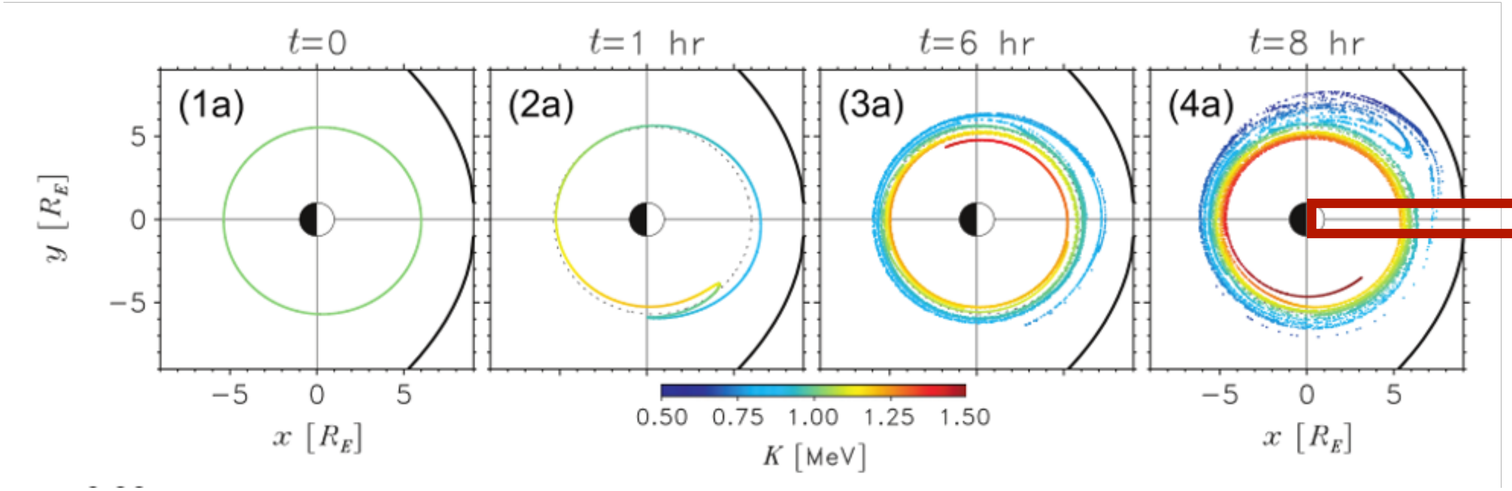
Radial diffusion. (Roederer & Zhang) Under a short kick at Q, one of the particles has moved to a different drift orbit. As it's only a shortlived kick, this is not true of the rest of the particles. Note the kick can be azimuthal _or_ radial.

Example: Radial diffusion



Numerical simulation, radial diffusion via ULF waves, Ukhorskiy & Sitnov 2008.

Example: Radial diffusion



Azimuthal variation is collapsed onto a radial co-ordinate *L*; we average over each drift.

Numerical simulation, radial diffusion via ULF waves, Ukhorskiy & Sitnov 2008.

- The diffusion equation for particle phase space density *f* under radial diffusion:
- Note that this is in a radial co-ordinate system.
- All the physics is contained in D_{LL}.

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\bigg|_{\mu,J} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L}\bigg|_{\mu,J} \right)$$

L ~ "Distance" of drift orbit,
D_{LL} ~ Diffusion coefficient (effect of electromagnetic perturbations)

$$D_{LL} = \frac{1}{2} \langle (\Delta L)^2 \rangle$$

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Uncertainty in complex systems

Uncertainty that is inherent or due to modelling → ensemble modelling / data assimilation → in space weather models.

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Why ensemble modelling?

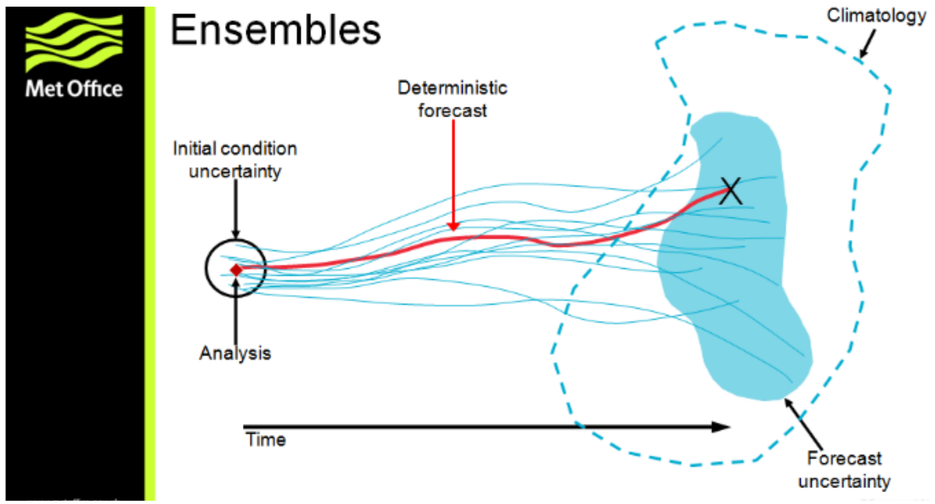
(For space weather generally)

- Running ensembles over many slightly varied inputs such as initial conditions:

  A. Gives an uncertainty associated to any modelling.

  B. Represents the inherent physical variability in the radiation belts.

- Improved modelling!



(Met Office) Ensembles can be used to capture uncertainty that reflects physical reality.

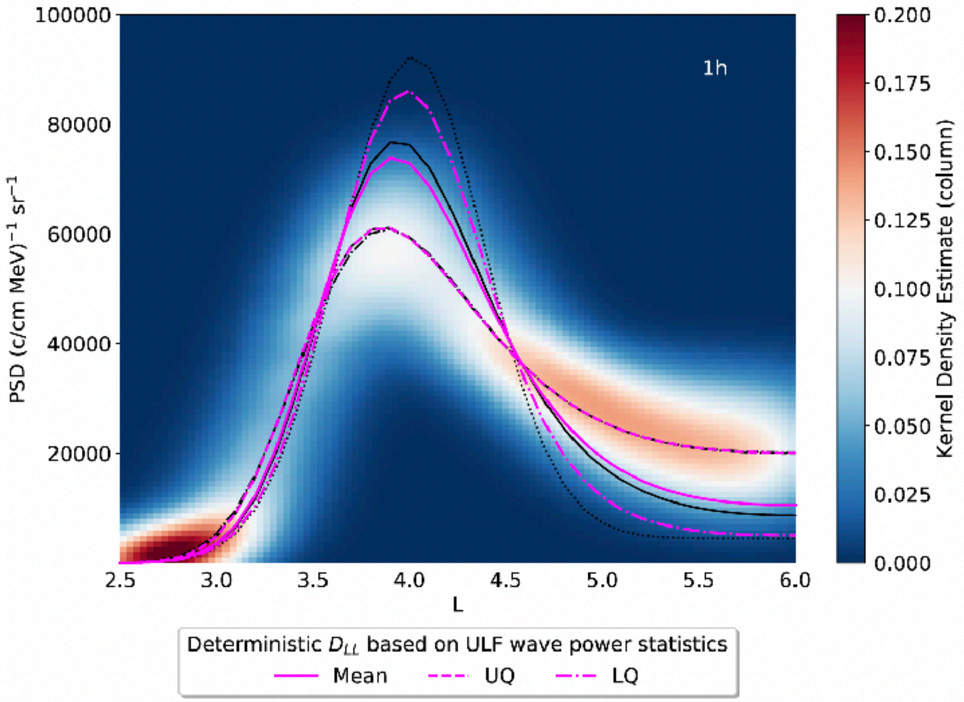
Why ensemble modelling?

(For radial diffusion)

- Huge variability in the radiation belts anyway
  → huge uncertainty to represent

- [Thompson+ 2020] showed that variability in the amount of diffusion gives us different final electron distributions; ensemble modelling is needed.

- [Bentley+ 2024] Benchmarking radial diffusion timescales ready for ensemble modelling.



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How to use ensemble models for the radiation belts?

Run many experiments → make metrics to compare them → realise some of our approaches may be fundamentally flawed (?!)

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How to use ensemble models for the radiation belts?

Run many experiments → make metrics to compare them → realise some of our approaches may be fundamentally flawed (?!)

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Methodology

Numerical Experiments Thompson et al., 2020

- Diffusion equation

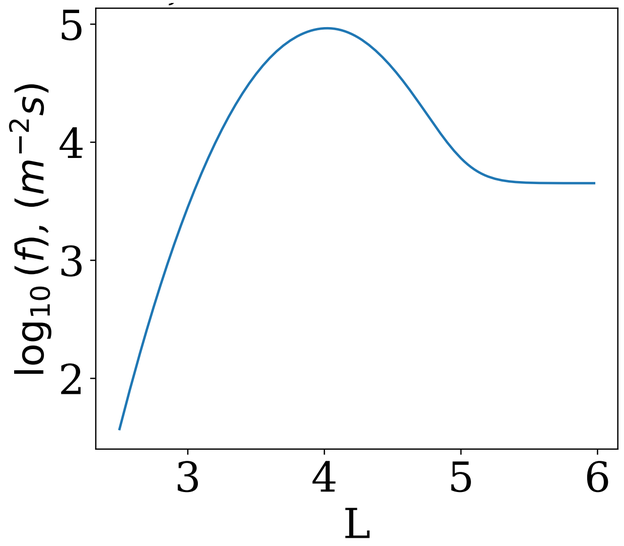
$$\frac{\partial f}{\partial t} = L^2 \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \mathcal{L} f$$

$$\mathcal{L} = \begin{cases} \frac{1}{\tau}, & \text{if } L \leq L_p \\ 0, & \text{otherwise} \end{cases}$$

Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007

$L \sim$ "Distance" of drift orbit, $D_{LL} \sim$ Diffusion coefficient (effects of electromagnetic perturbations)

D_{LL} determines the amount of radial diffusion, and depends on L



- Initial phase space density (PSD) distributions f: background particles and an enhancement

Bentley et al. 2024, Physics of Plasmas

Methodology

Numerical Experiments Thompson et al., 2020

$$D_{LL} \propto L^8 10^{aL+bKp^2+cKp}$$

Ozeke et al., 2014

$$D_{LL} \propto L^n, n = 8$$

- Diffusion equation

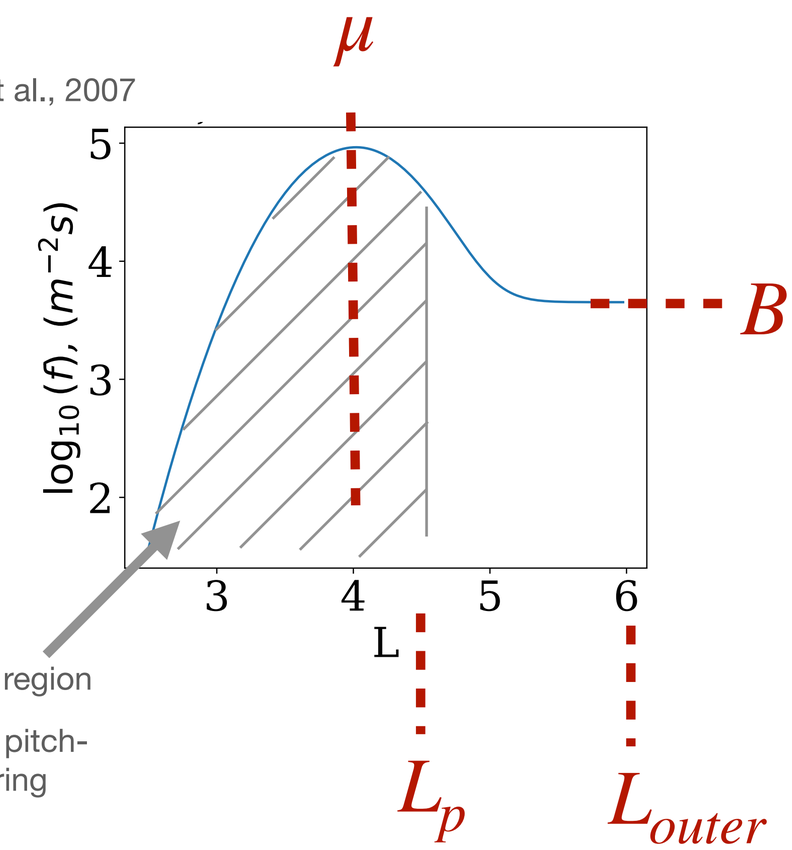
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Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007



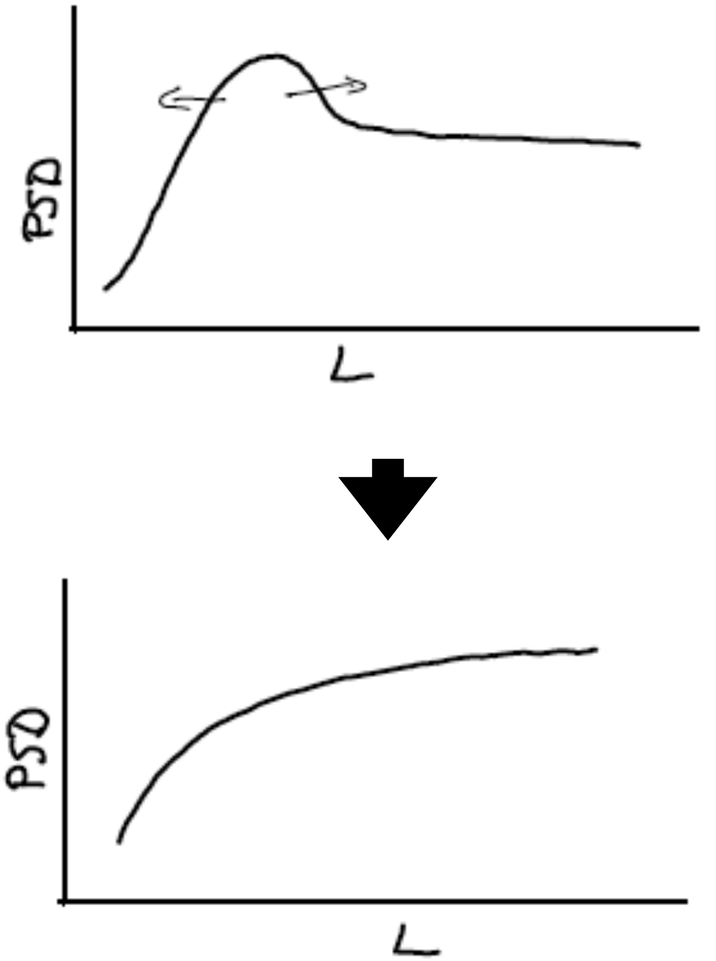
Loss $\frac{1}{\tau} f$ in this region to approximate pitch-angle scattering

- Initial phase space density (PSD) distributions f: background particles and an enhancement

Metric 1: Time to monotonicity t_m

"How long until the enhancement has diffused away?"

- Can't use MSE etc because then you need a threshold for when two PSD distributions are "close enough" → your results become threshold-dependent
- Represents the physics we are often interested in using the shape of the phase space density (PSD) distribution.



Metric 2: Analytical approach $\mathscr{E}, \mathscr{N}$

Mass- and energy-like density

$\mathcal{N}, \mathcal{E}$ (Mass- and energy-like density)

$$\mathcal{N} = \int f \frac{1}{L^2} dL$$

Mass in experiment

$$\mathcal{E} = \frac{1}{2c_{\mathcal{E}}} \int |f|^2 \frac{1}{L^2} dL$$

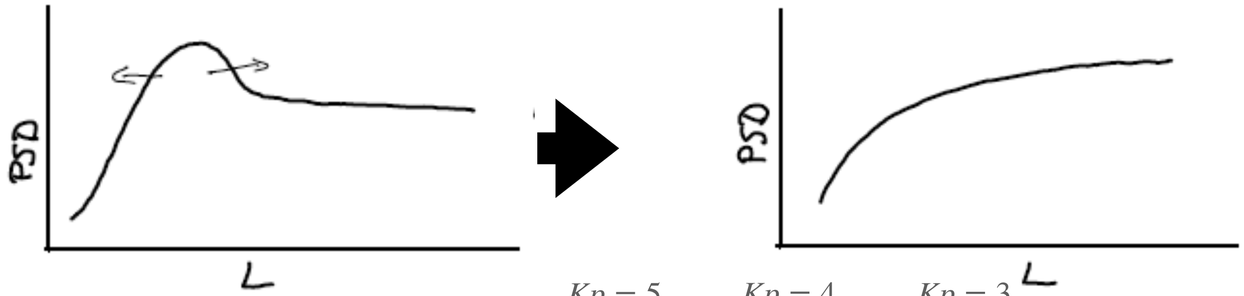
How close the distribution is to a zero state; or a state that does not change with time

t_m tells us…

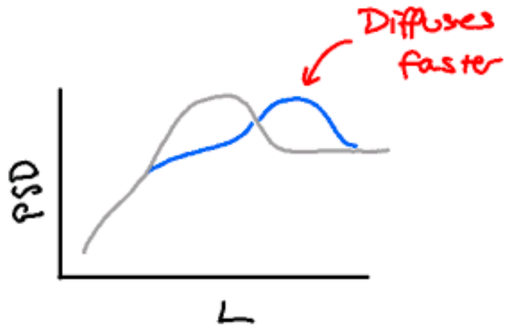
Diffusion varies with μ, B, L_{outer} and outer boundary condition

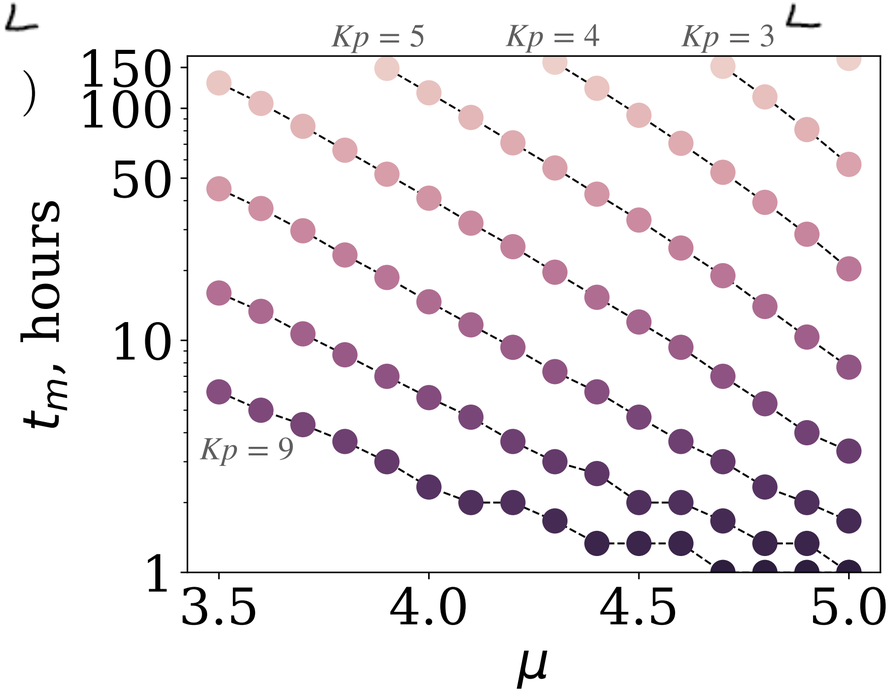
Time to monotonicity t_m

"How long until the enhancement has diffused away?"



Example: enhancement location μ





t_m varying with μ and Kp, with a Neumann (constant gradient) outer boundary condition.

t_m tells us…

Diffusion varies with μ, B, L_{outer} and outer boundary condition

Time to monotonicity t

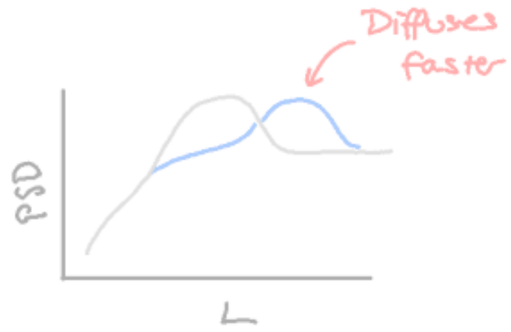
"How lo
has diff

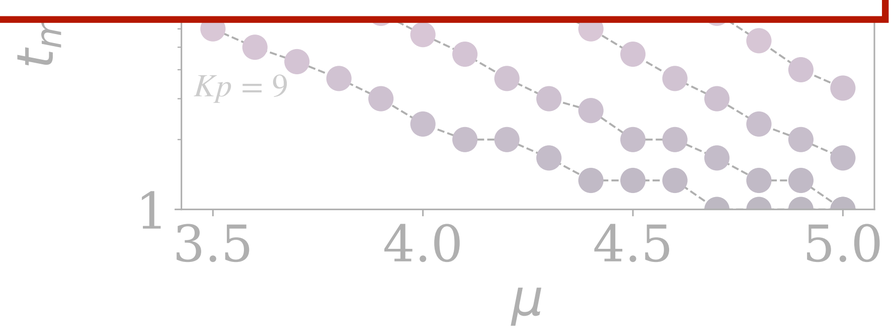
Exampl

Recall that

$$D_{LL} \propto L^8 10^{aL+bKp^2+cKp}$$

i.e. diffusion coefficient varies depending on where we are in L!

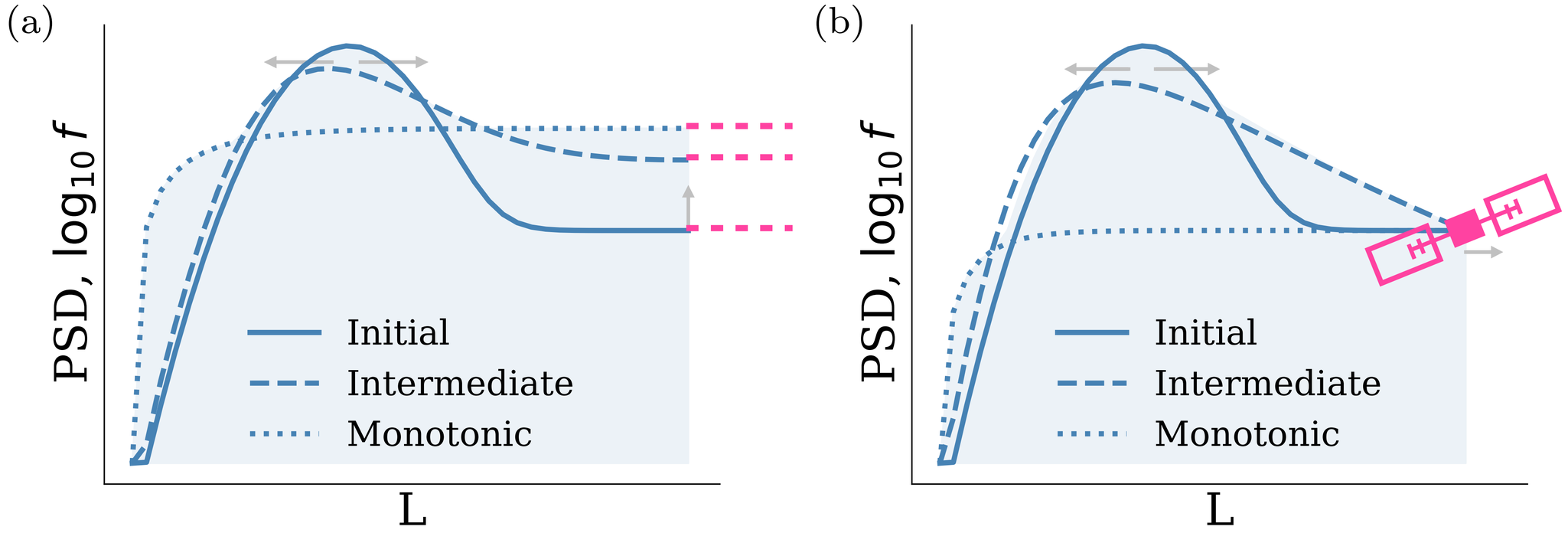




t_m varying with μ and Kp, with a Neumann (constant gradient) outer boundary condition.

t_m tells us…

The outer boundary condition changes radiation belt behaviour



Neumann outer boundary (variable value; zero flux)

Dirichlet outer boundary (constant value)

The enhancement diffuses differently depending on the outer boundary; which determines whether the enhancement can diffuse into high L.

How significant are these effects in reality?

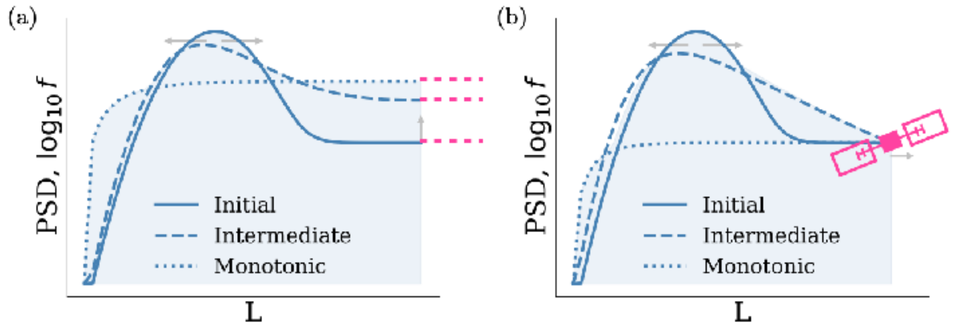
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- Do we care about initial conditions compared to loss from pitch-angle scattering?

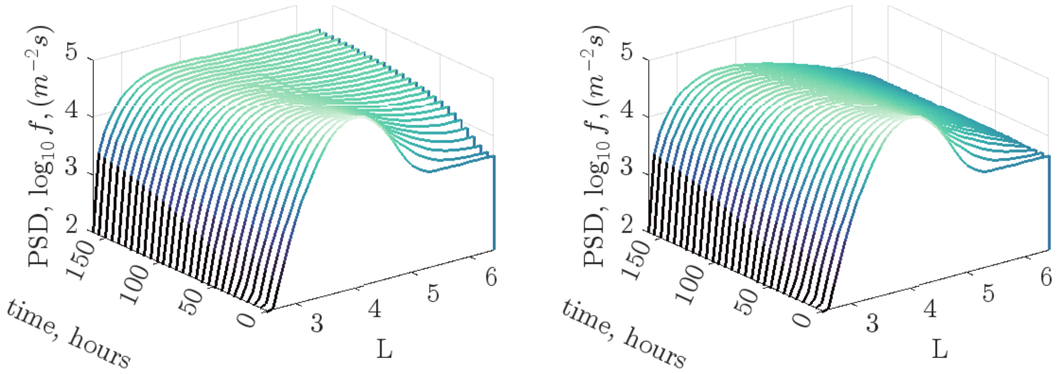
Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007

- Both outer boundary conditions shown previously are physical and unphysical in different ways.



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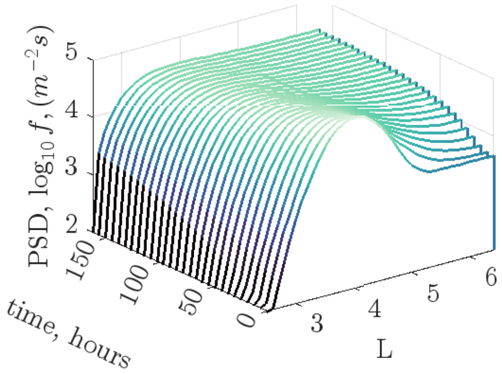
These effects are dominated by loss.

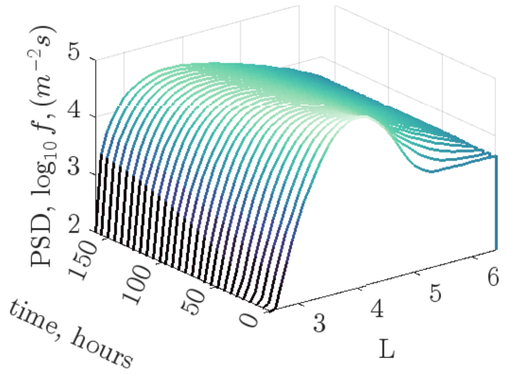


Starting with a different outer boundary condition and no loss results in a different evolution.

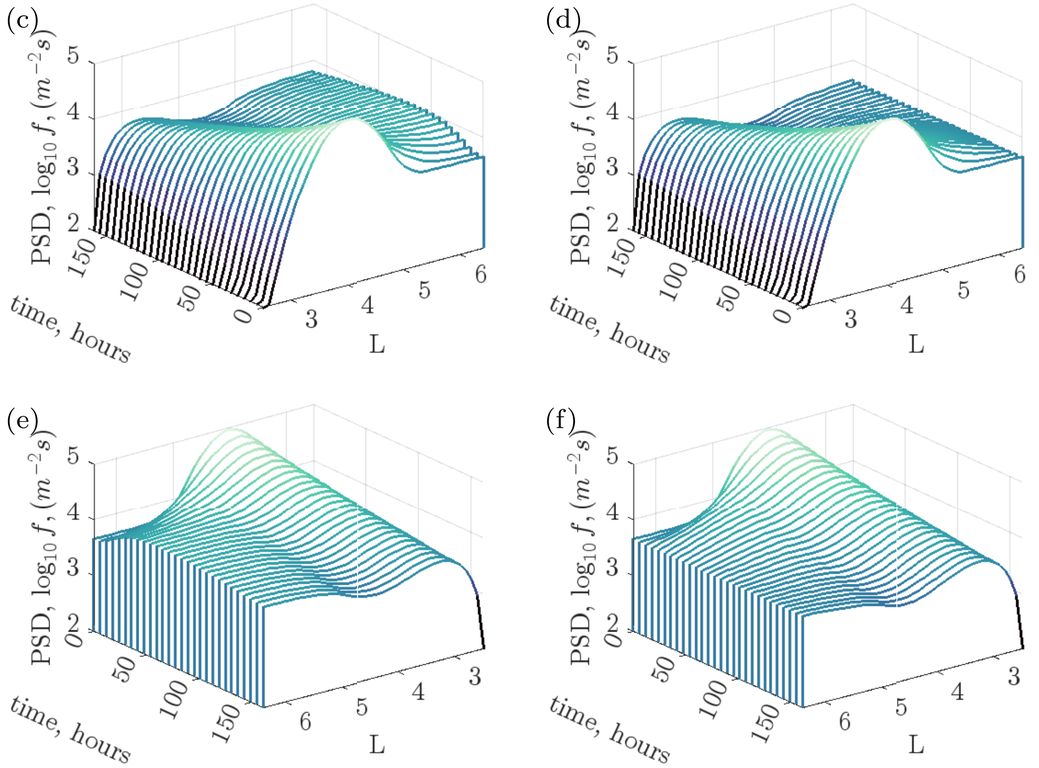
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These effects are dominated by loss.





Starting with a different outer boundary condition and no loss results in a different evolution.



Same simulations, with loss: The evolution appears much less boundary-condition dependent.

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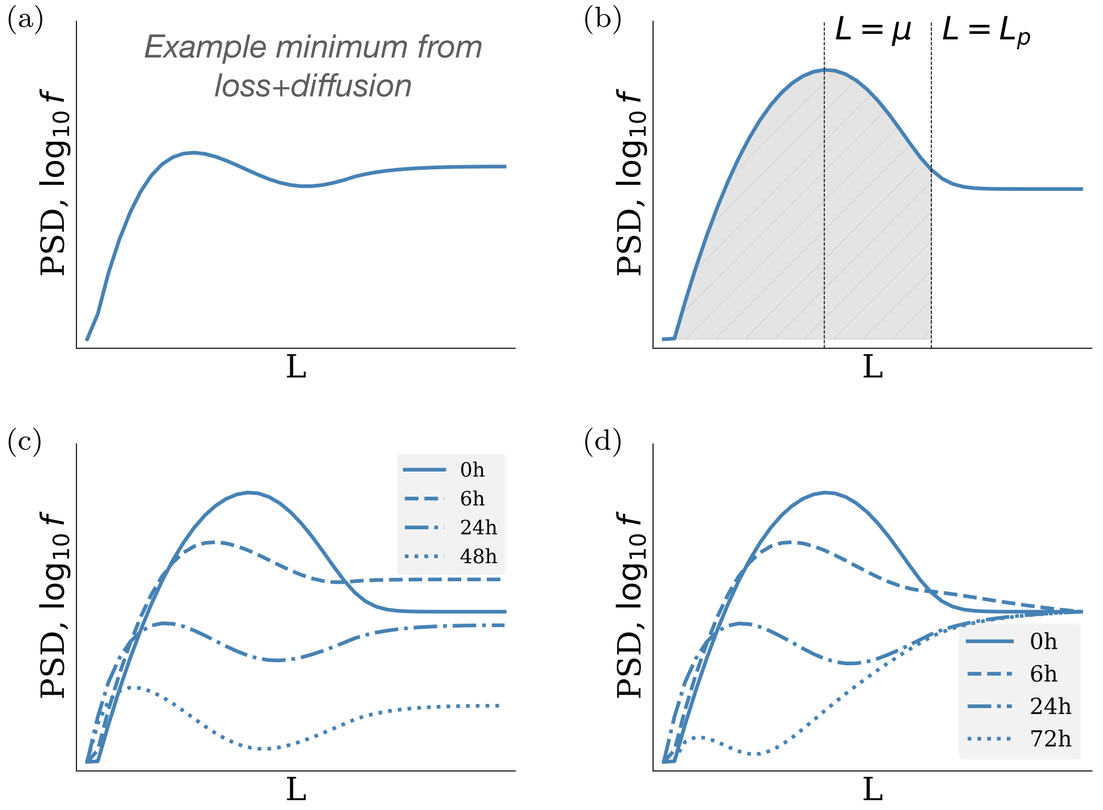
How does loss affect our ensemble?

Plasmapause location vs μ is important

- If we add loss, then there is at least one new parameter to benchmark: the edge of that loss, the plasmapause.
- The location of the plasmapause compared to the enhancement can result in monotonicity never being reached: t_m would need to be adapted in this case.

Explanatory doodle:

Plasmapause location is higher L than enhancement



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Metric 2: Mass- and energy-like density $\mathscr{N}$, $\mathscr{E}$

We can quantify how f moves towards equilibrium; how it diffuses

$\mathscr{N}$, $\mathscr{E}$ (Mass- and energy-like density)

$$\mathcal{N} = \int f \frac{1}{L^2} dL$$

Mass in experiment

$$\mathcal{E} = \frac{1}{2c_{\mathcal{E}}} \int |f|^2 \frac{1}{L^2} dL$$

How close the distribution is to a zero state; or a state that does not change with time

Metric 2: Mass- and energy-like density $\mathscr{N}$, $\mathscr{E}$

We can quantify how f moves towards equilibrium; how it diffuses

$\mathscr{N}$, $\mathscr{E}$ (Mass- and energy-like density)

$$\mathcal{N} = \int f \frac{1}{L^2} dL$$

Mass in experiment

$$\mathcal{E} = \frac{1}{2c_{\mathcal{E}}} \int |f|^2 \frac{1}{L^2} dL$$

How close the distribution is to a zero state; or a state that does not change with time

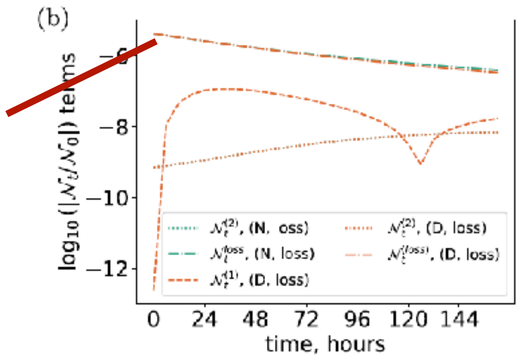
Using $D_{LL} = D_0 L^n$ we can exactly specify how the boundary conditions and PSD contribute to changes in $\mathscr{N}$, $\mathscr{E}$ over time, e.g.

$$\mathcal{E}_t = \left[\frac{D_{LL}}{L^2} f f_L \right]_{IB}^{OB} - \int \frac{D_{LL}}{L^2} (f_L)^2 \, dL$$

How *f* and DLL affect the rate the PSD trends toward a minimum energy state.

Example uses: Track the changing mass, track what conditions contribute the most to the changing PSD *f*

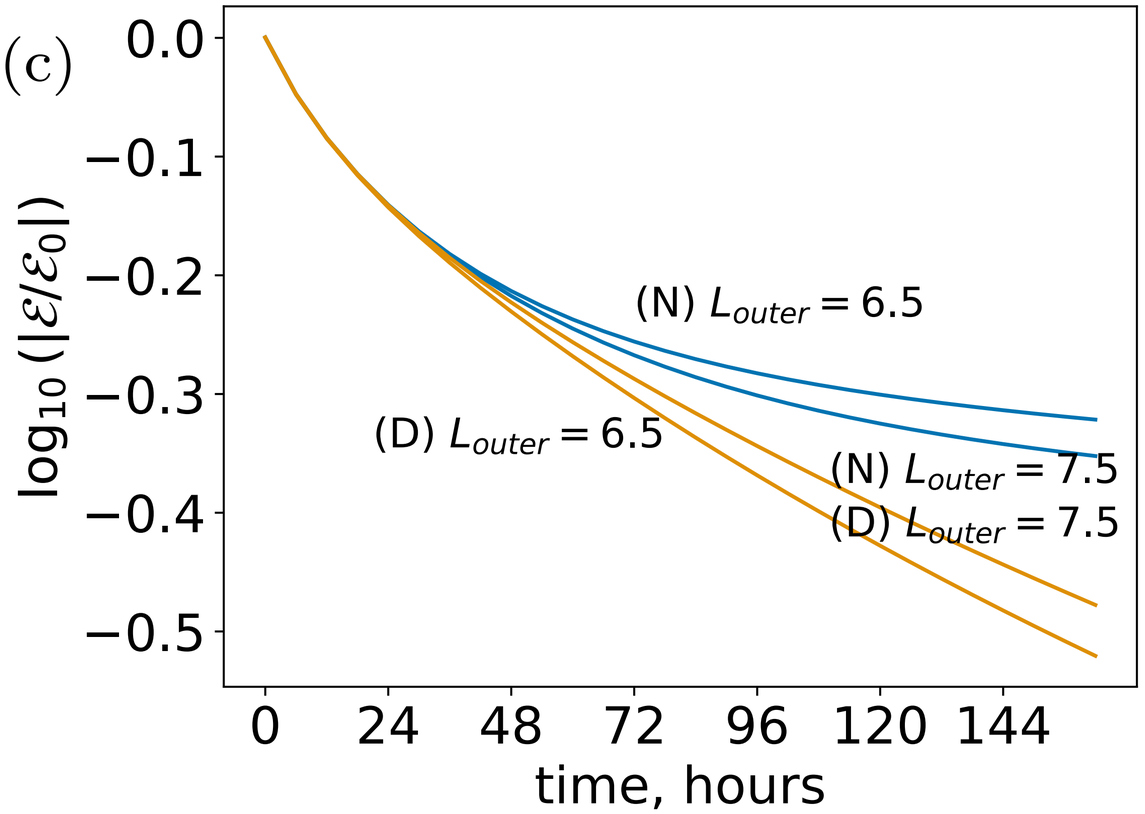
Calculating all components of $\mathscr{N}_t$, the biggest component is loss; loss contributes the most to the changing particle density.



$\mathscr{E}$ over a week

Outer boundary condition and location matters

- $\mathscr{E}$ over a week for two different outer boundary conditions and two different outer boundary locations.
- Remember: the lower the $\mathscr{E}$, the closer we are to to low "energy"/ equilibrium state

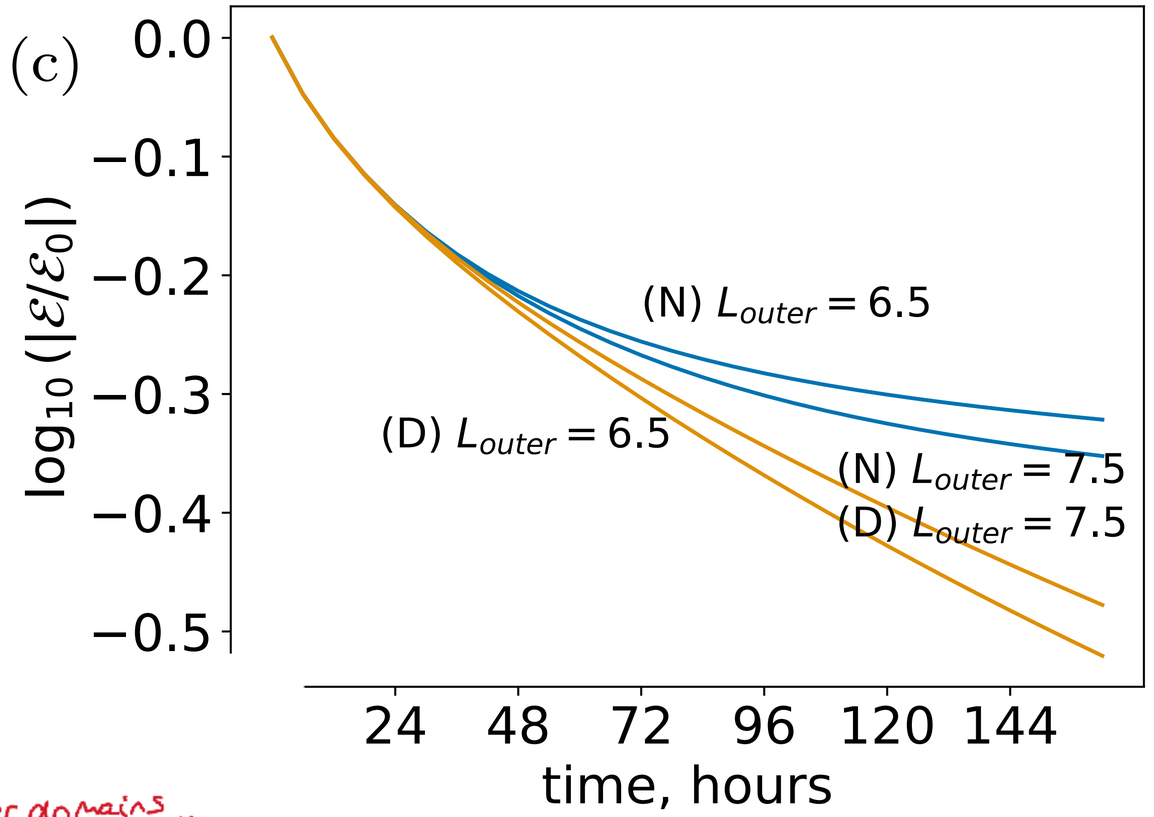


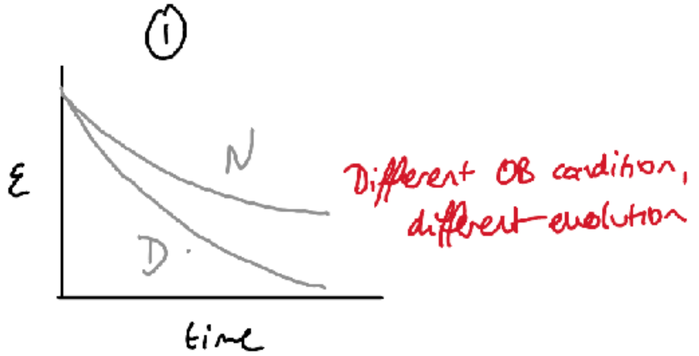
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$\mathscr{E}$ over a week

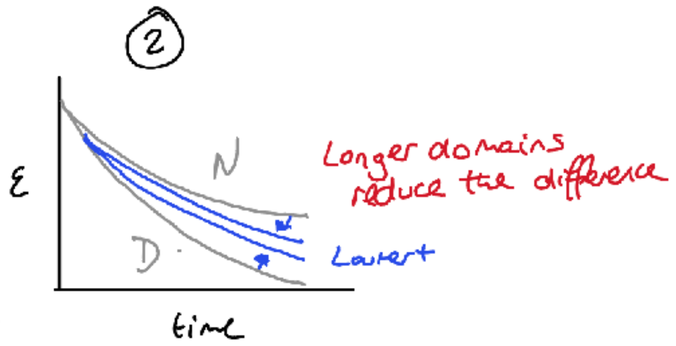
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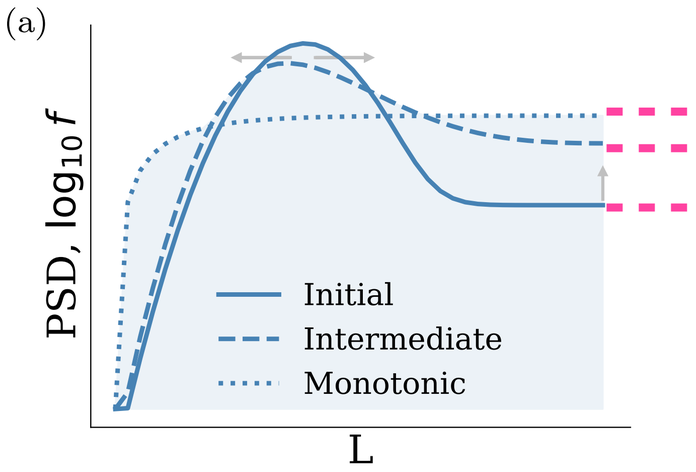


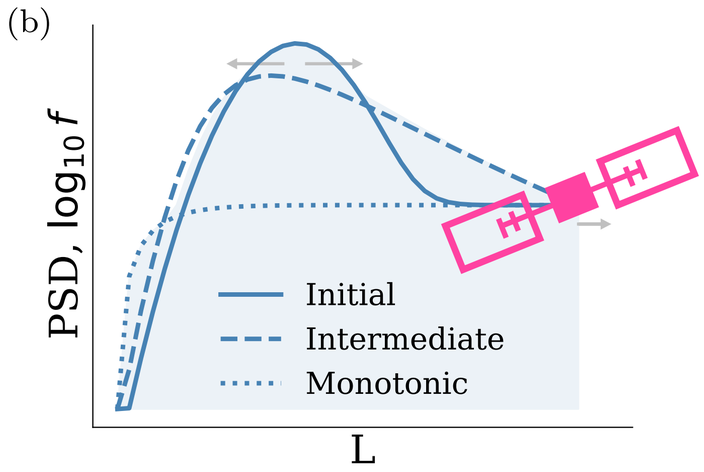


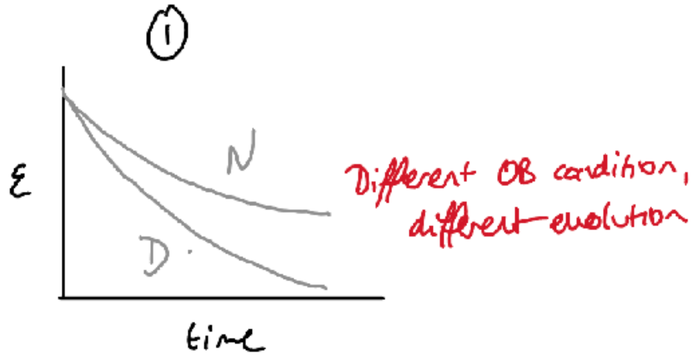
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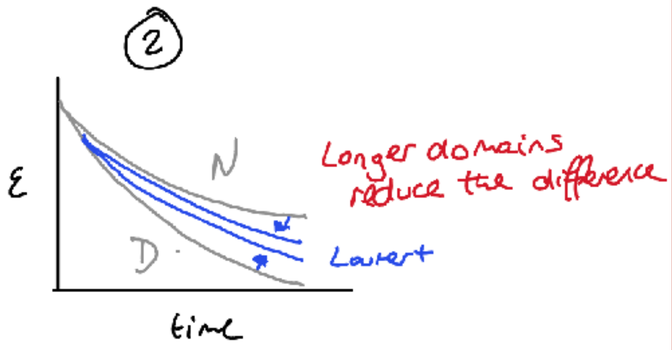


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Earlier: both boundary conditions could impose unphysical processes.

Now: the length of the domain ***also*** changes the evolution.

- Cutting short a domain to where there is data will remove some of the physics.
- Lee+ 2024 showed that a shorter domain with data is better.
- This leaves big questions for forecasting, when data is unavailable.

Really, we need outer boundary **location**, **value** (data!) and different (mixed?) boundary **conditions**.

Bentley et al. 2024, Physics of Plasmas

Gradients > D_{LL}

$\mathscr{E}_t$ allows us to test how D_{LL} vs f_L contribute to evolution of PSD f

- Typical diffusion "acts to reduce gradients" as

$$\mathscr{E}_t = -\int (f_x)^2 dx$$

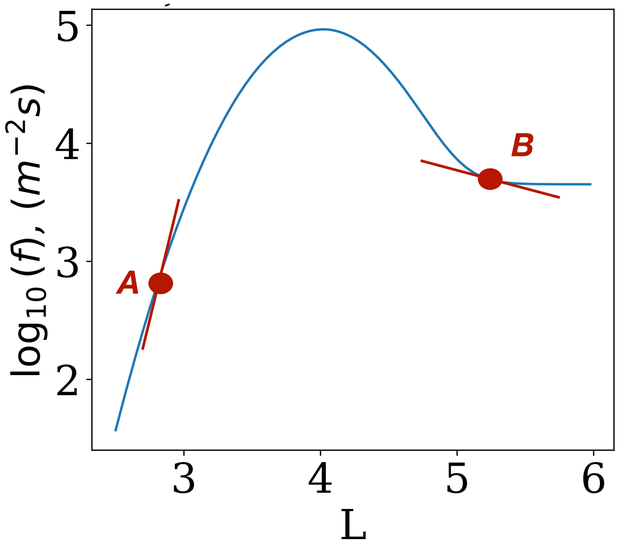
- Using $D_{LL} = D_0 L^n, n = 8$, we find

$$\mathscr{E}_t = \left[\frac{D_{LL}}{L^2} f f_L\right]_{IB}^{OB} - \int \frac{D_{LL}}{L^2}(f_L)^2 dL$$

- Radial diffusion is more complicated!

Typical, constant-D diffusion:

Steeper gradient at A therefore more diffusion than at B



Radial diffusion:

What about D_{LL}?

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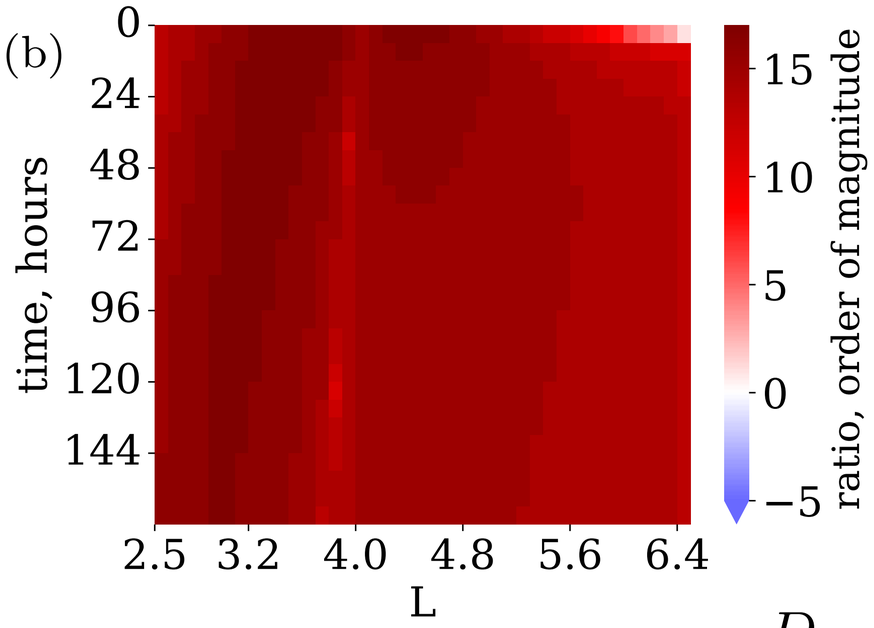
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- Radial diffusion is more complicated!

But which of D_{LL}, f_L dominate the system evolution?



Red means: $(f_L)^2 > \frac{D_{LL}}{L^2}$

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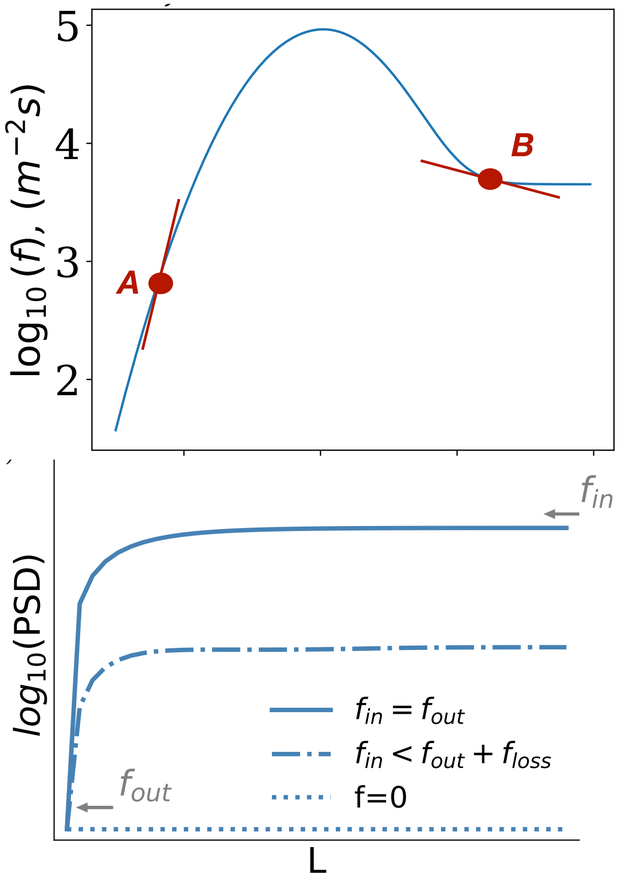
Typical, constant-D diffusion:

Steeper gradient at A therefore more diffusion than at B

Radial diffusion:

The same!

We end up with monotonic states that are extremely skewed to low L



Three example final, monotonic distributions.

Gradients > D_{LL}

What does this mean?

- Conventional wisdom is the opposite: that in the radiation belts, $D_{LL} > f_L$.
- If this is wrong, should we be spending our effort determining f (and hence f_L) over D_{LL}?
- OR are our results wrong? Is the 1D radial diffusion model too simplistic, and we need the other two types of diffusion?

Conclusions

1. Outer boundary condition and location change the system evolution, and it's not clear what we should use.
   - for now, using data constrains the simulation but this is not available for forecasting.
2. Gradients may be contributing more to diffusion than diffusion coefficients.
   - EITHER we need to shift our focus as a field OR stop using 1D radial diffusion.
3. We developed an intuitive metric (t_m) and a mathematically sound one ($\mathscr{E}, \mathscr{N}$)

Northumbria University NEWCASTLE

Bentley et al. 2024, Physics of Plasmas; doi:10.1063/5.0217268

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Numerical Experiment setup

Thompson et al., 2020

- Diffusion equation

  (We run with, and without loss)

$$\frac{\partial f}{\partial t} = L^2 \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \mathcal{L} f$$

$L \sim$ "Distance" of drift orbit, $D_{LL} \sim$ Diffusion coefficient (effects of electromagnetic perturbations)

$$\mathcal{L} = \begin{cases} \frac{1}{\tau}, & \text{if } L \leq L_p \\ 0, & \text{otherwise} \end{cases}$$

Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007

- Diffusion coefficients

$$(1) \quad D_{LL} = D_{LL}^B + D_{LL}^E$$

$$(2) \quad D_{LL} = D_0 L^n, n = 6$$

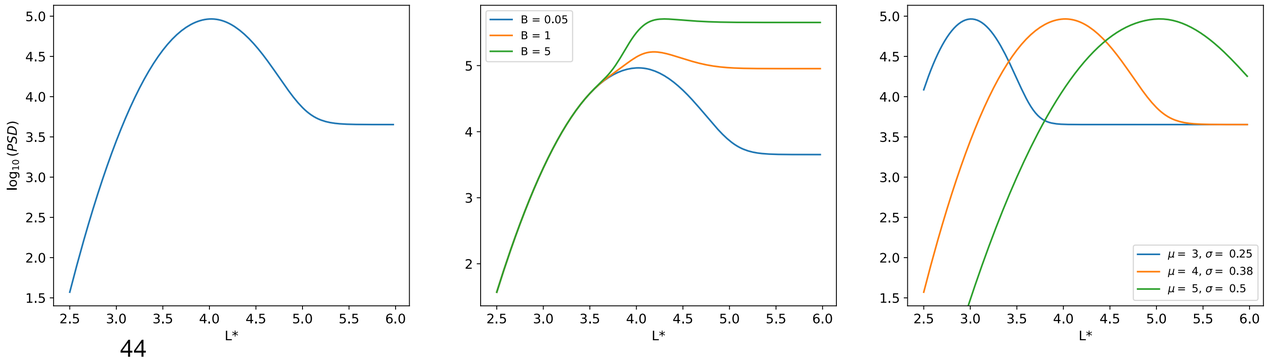
$$D_{LL}^E = 2.16 \times 10^{-8} L^6 10^{0.217L + 0.461Kp}$$

$$D_{LL}^B = 6.62 \times 10^{-13} L^8 10^{-0.0327L - 0.108K^2 p + 0.499Kp}$$

Diffusion coefficient parameterisation from Ozeke et al., 2014

- Initial phase space density (PSD) distributions: background particles and an enhancement

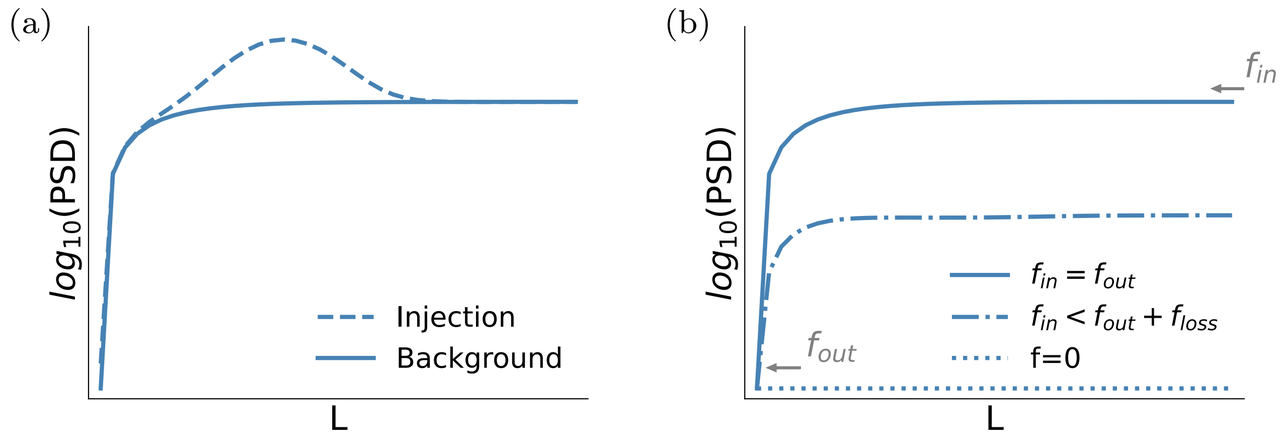
$$f = A \exp\left(-\frac{(L-\mu)^2}{2\sigma^2} \right) + \frac{1}{2} AB \left[erf(\gamma(L-\mu)) + 1 \right]$$



Bentley et al. 2024, Physics of Plasmas

Metric 1: Time to monotonicity *tm*

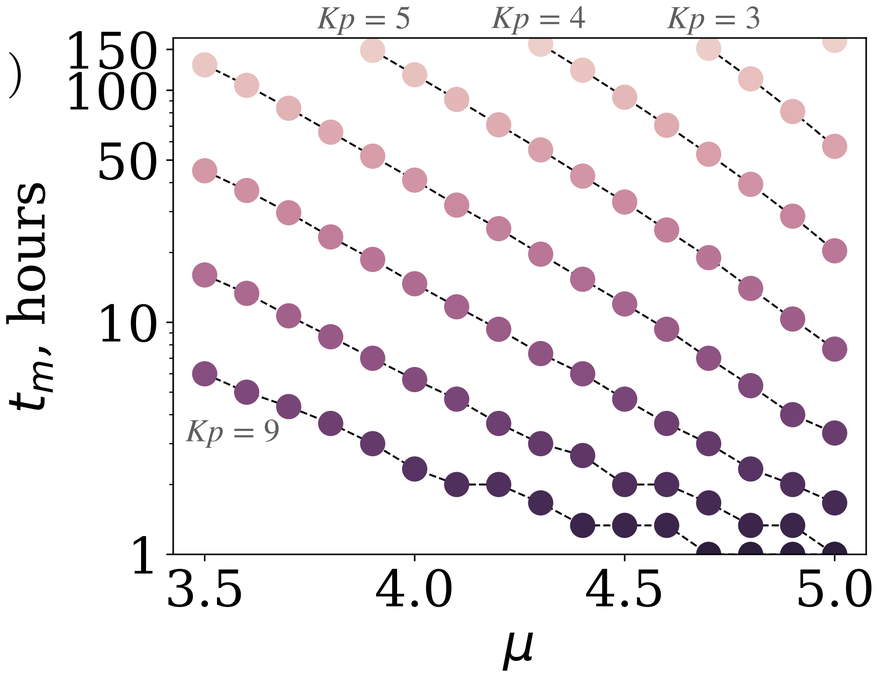
- Can't use MSE etc because then you need a threshold for when two PSD distributions are "close enough" → your results become threshold-dependent
- We look at the interesting morphology: "how long for an enhancement to diffuse away"



Bentley2024+ Example phase space density (PSD) distributions.
(a) monotonic, "quiet/background" distribution + enhancement
(b) potential long term (monotonic) PSD profiles: a return to the quiet profile (solid), a reduced amplitude profile (dot-dashed) and a zero profile (dotted).

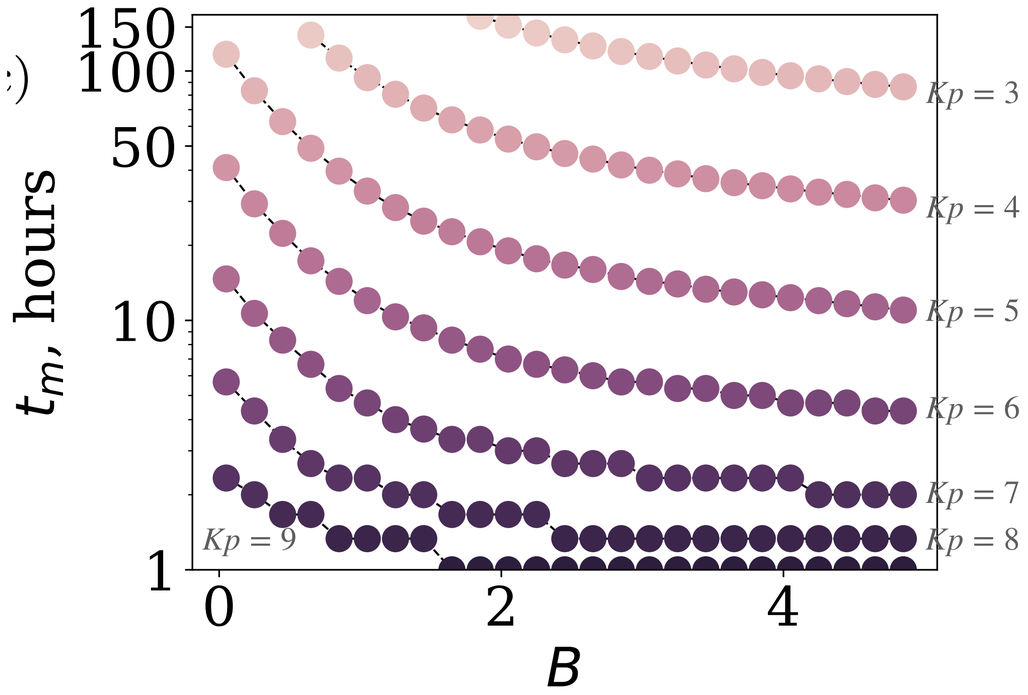
Selected results with t_m

Enhancement location μ and step size B



t_m varying with μ and Kp, with a Neumann (constant gradient) outer boundary condition.

With geomagnetic activity (Kp) held constant, the higher in L the enhancement is located the faster it diffuses away.



$t_m$varying with step size B and Kp, with a Neumann (constant gradient) outer boundary condition.

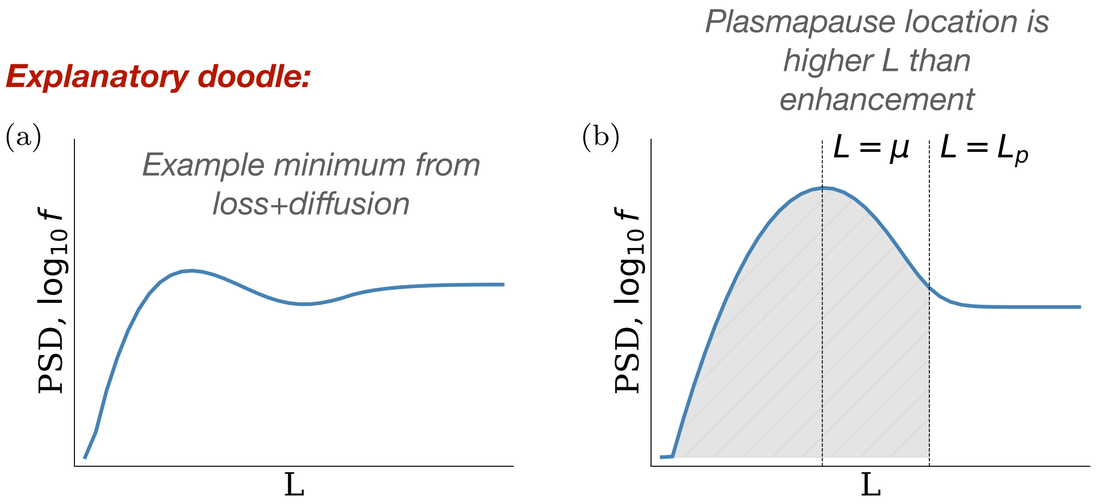
With geomagnetic activity (Kp) held constant, the higher the background population at the outer boundary, the quicker the population reaches monotonicity.

How does loss affect our ensemble?

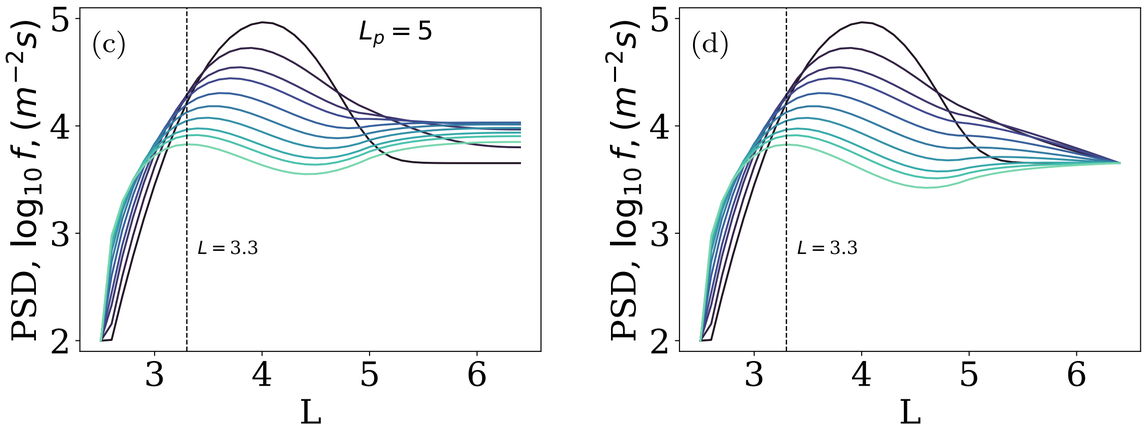
Plasmapause location vs μ is important

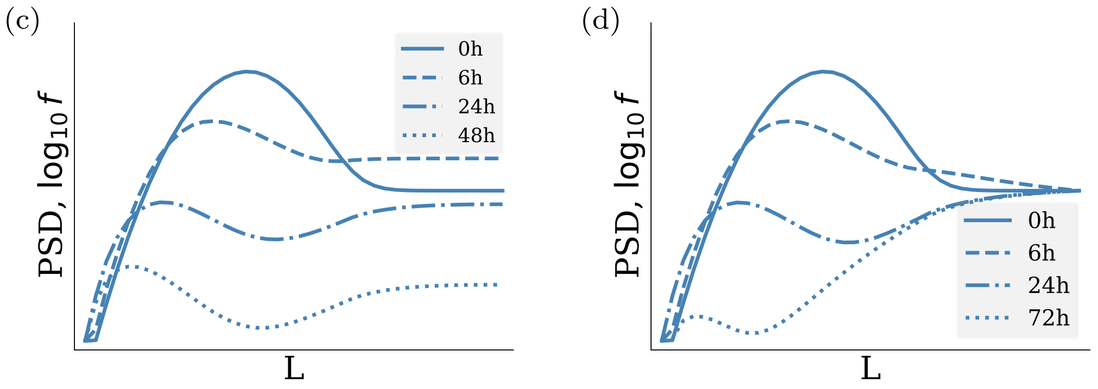
- Effects of outer boundary condition and location are dominated by loss - these are secondary, phew!
- The location of the plasmapause compared to the enhancement can result in monotonicity never being reached: t_m would need to be adapted in this case.

Explanatory doodle:



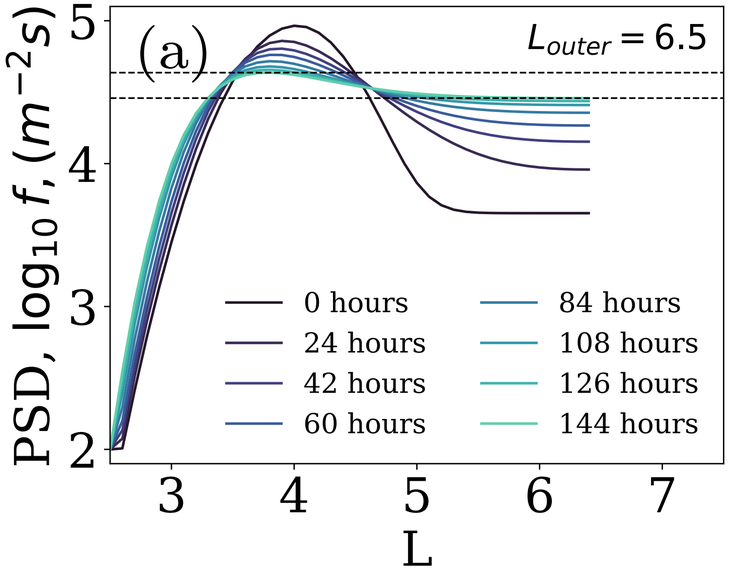
Simulations:

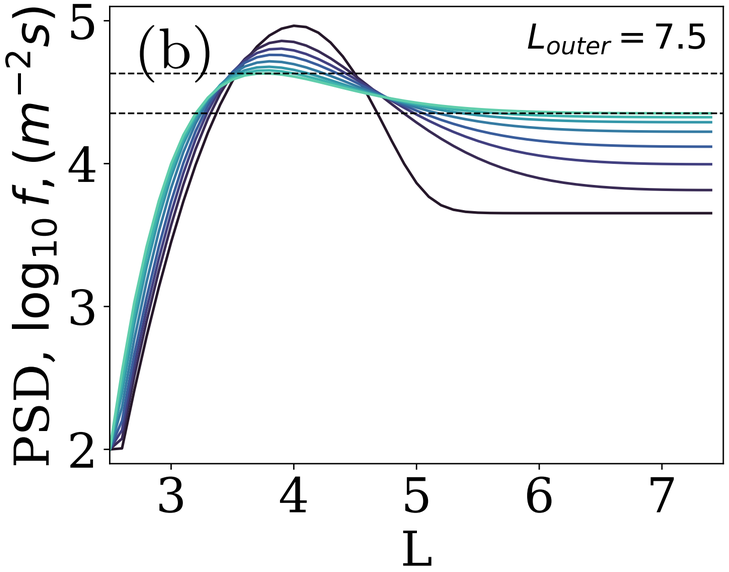


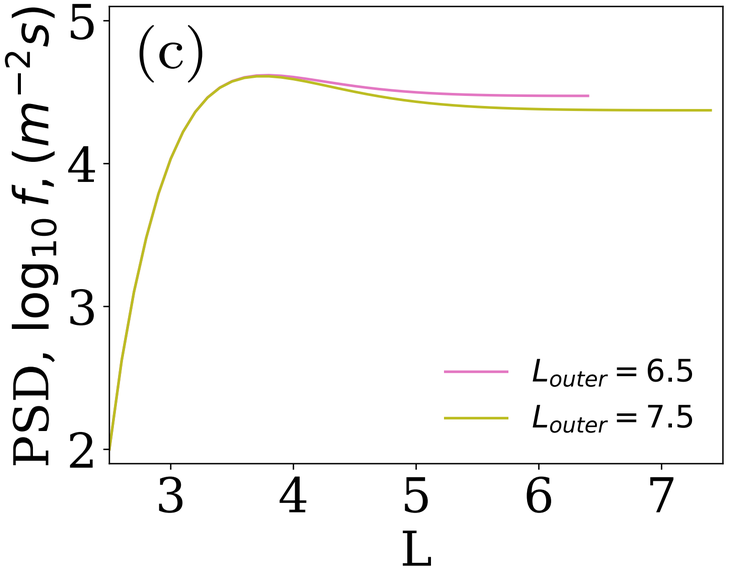


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What else about the outer boundary?







Numerical Experiment setup

We vary and examine:

- Strength of diffusion coefficient (via Kp)
- Initial distribution properties:
  - Amplitude A
  - Location μ of enhancement in L^*
  - Width σ of enhancement
  - “Background” particle population (step size B)
- Outer boundary location L_{outer}
- Outer boundary numerical condition (OBC).
- Loss $\mathscr{L}$, e.g. from pitch-angle scattering.
- Plasmapause location L_p.

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Some are more interesting!

