Northumbria University NEWCASTLE

How do fundamental modelling choices affect radial diffusion in Earth's Radiation Belts?

J. R. Stout, R. L. Thompson, D. J. Ratliff, C. E. J. Watt

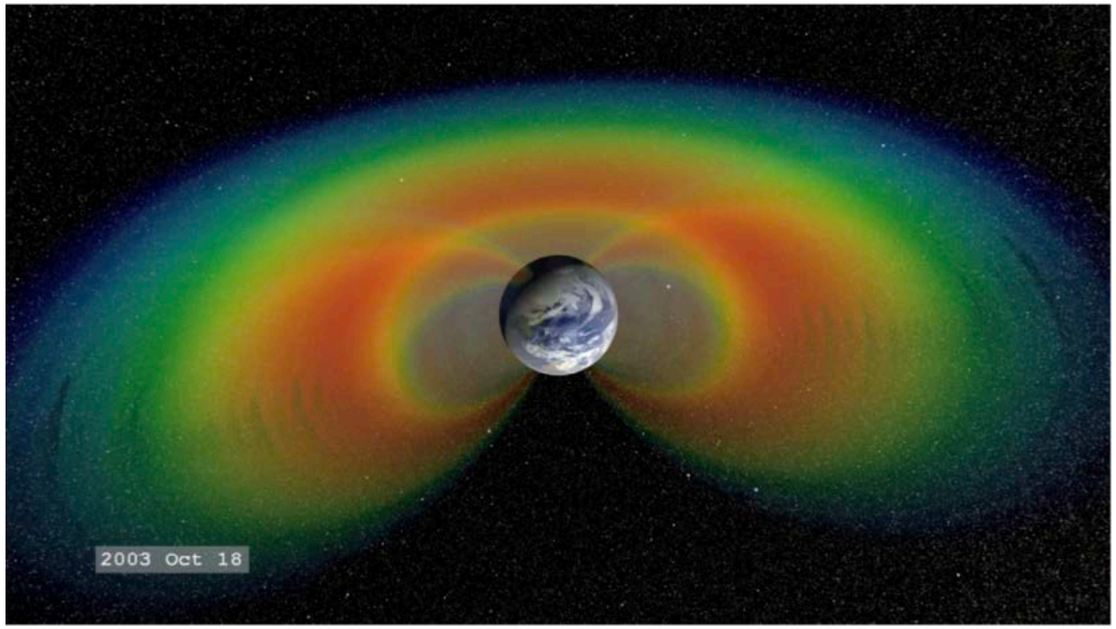
SN Bentley, Spring MIST, Leicester 2025

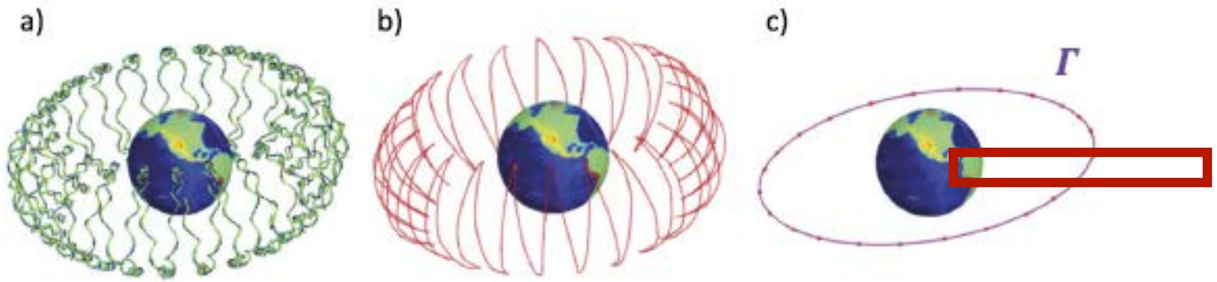
Bentley et al. 2024, Physics of Plasmas; https://doi.org/10.1063/5.0217268

Motivation

- Particles trapped by the Earth's magnetic field.
- Outer belt is highly variable, electrons, 3-10 R_E, a danger to spacecraft
- Three motions of gyro motion, bounce and **drift. Radial diffusion** is the diffusion of particles across drift paths.
- Assuming we want to improve space weather modelling by reducing uncertainty: use ensembles! How?

  ! Start with 1D radial diffusion, look at impact of model choices.





Three particle motions (Lejosne). So there are three conserved quantities, each one associates with the timescale of one periodic motion.

Methodology

Numerical Experiments Thompson et al., 2020

- Diffusion equation

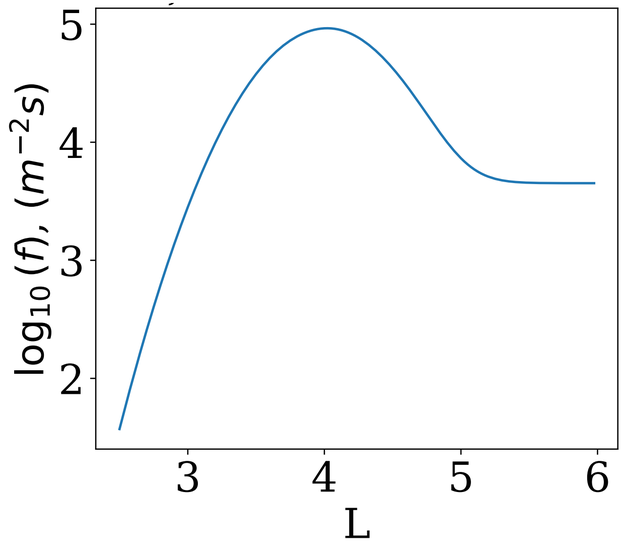
$$\frac{\partial f}{\partial t} = L^2 \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \mathcal{L} f$$

$$\mathcal{L} = \begin{cases} \frac{1}{\tau}, & \text{if } L \leq L_p \\ 0, & \text{otherwise} \end{cases}$$

Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007

$L \sim$ "Distance" of drift orbit, $D_{LL} \sim$ Diffusion coefficient (effects of electromagnetic perturbations)

D_{LL} determines the amount of radial diffusion, and depends on L



- Initial phase space density (PSD) distributions f: background particles and an enhancement

Bentley et al. 2024, Physics of Plasmas

Methodology

Numerical Experiments Thompson et al., 2020

$$D_{LL} \propto L^8 10^{aL+bKp^2+cKp}$$

Ozeke et al., 2014

$$D_{LL} \propto L^n, n = 8$$

- Diffusion equation

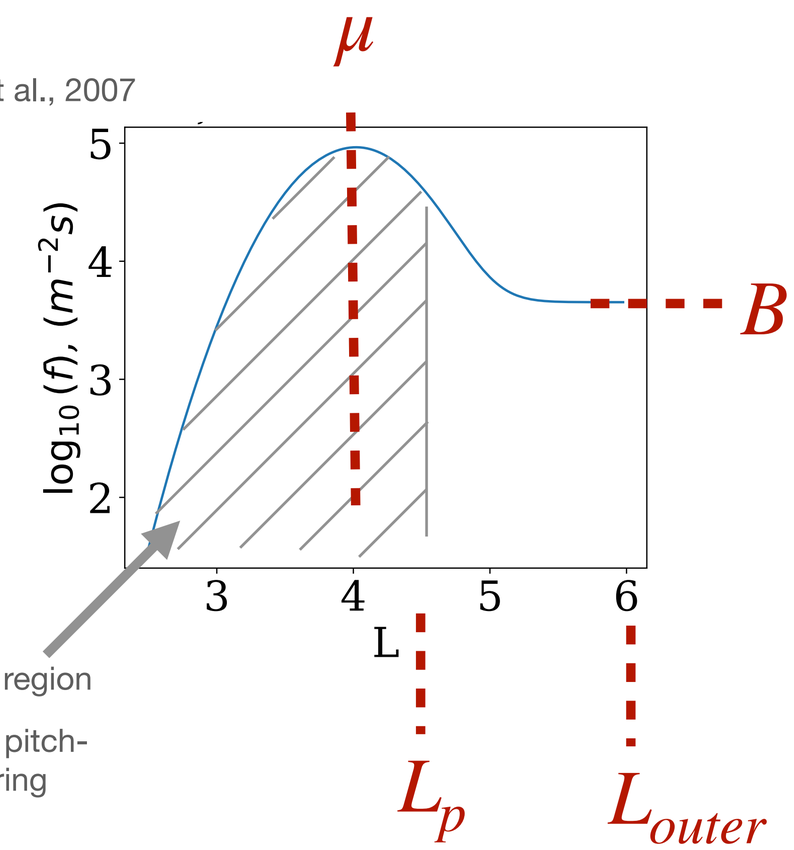
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Loss via pitch angle scattering
Electron timescale τ from Shprits et al., 2007



Loss $\frac{1}{\tau} f$ in this region to approximate pitch-angle scattering

- Initial phase space density (PSD) distributions f: background particles and an enhancement

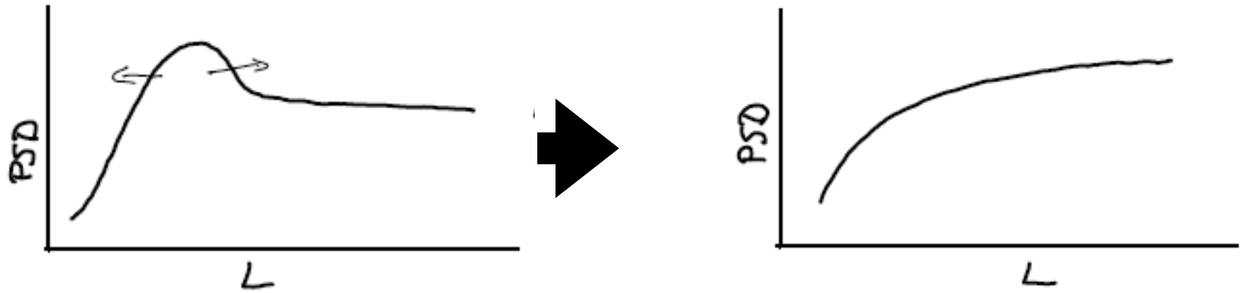
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Key results #1: t_m tells us…

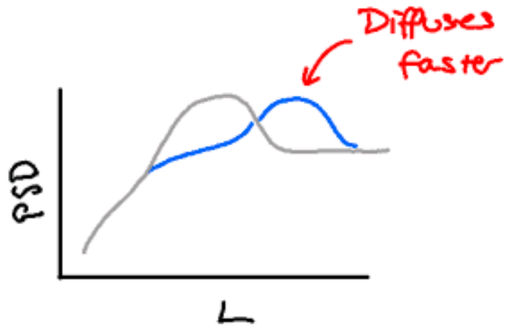
Diffusion varies with μ, B, L_{outer} and outer boundary condition

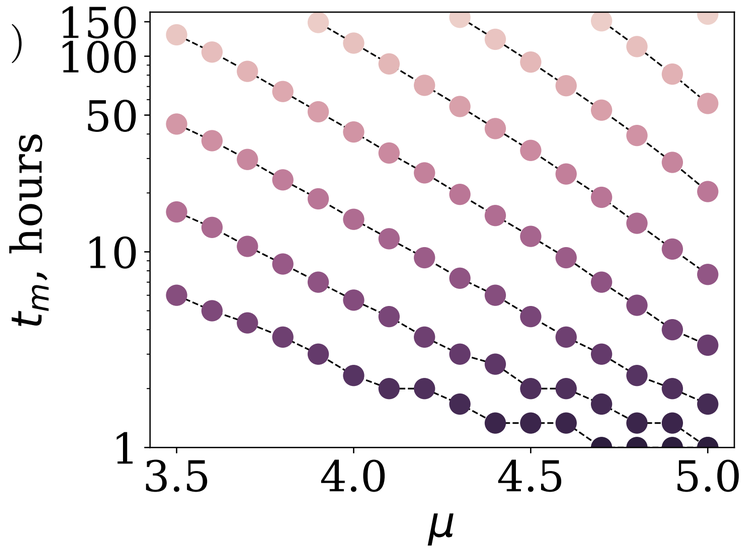
Time to monotonicity t_m

"How long until the enhancement has diffused away?"



Example: enhancement location μ

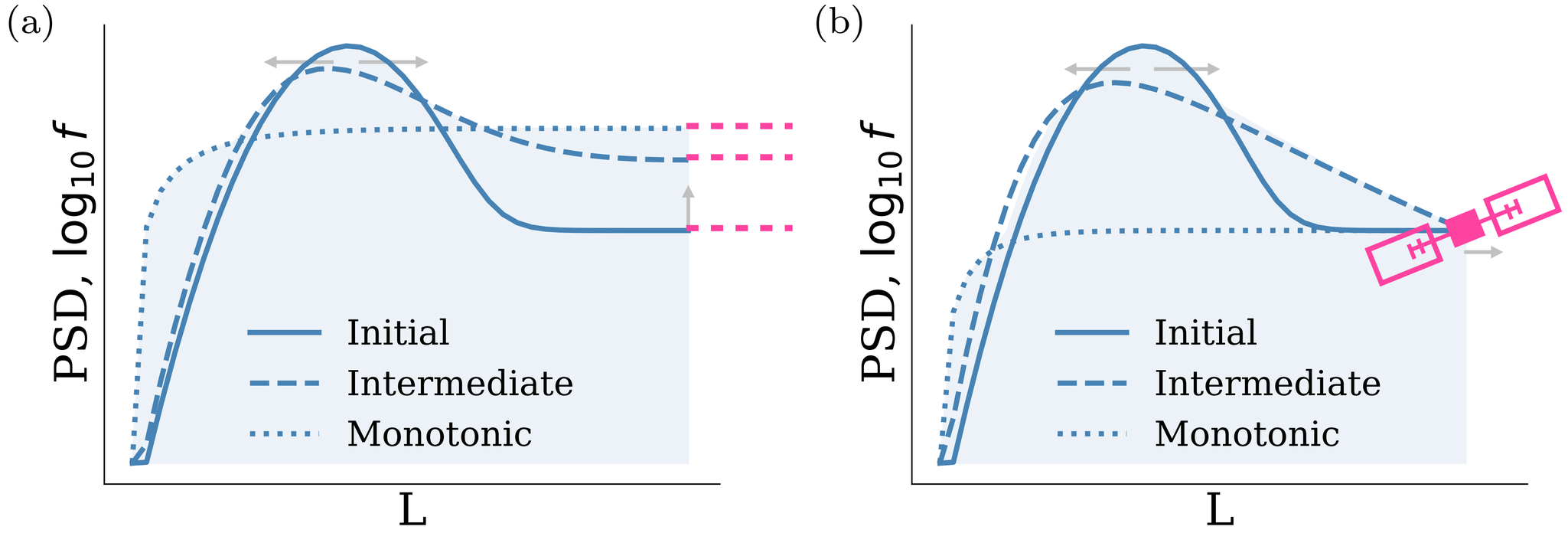




t_m varying with μ and Kp, with a Neumann (constant gradient) outer boundary condition.

Key results #2: t_m tells us…

The outer boundary condition changes radiation belt behaviour



Neumann outer boundary (variable value; zero flux)

Dirichlet outer boundary (constant value)

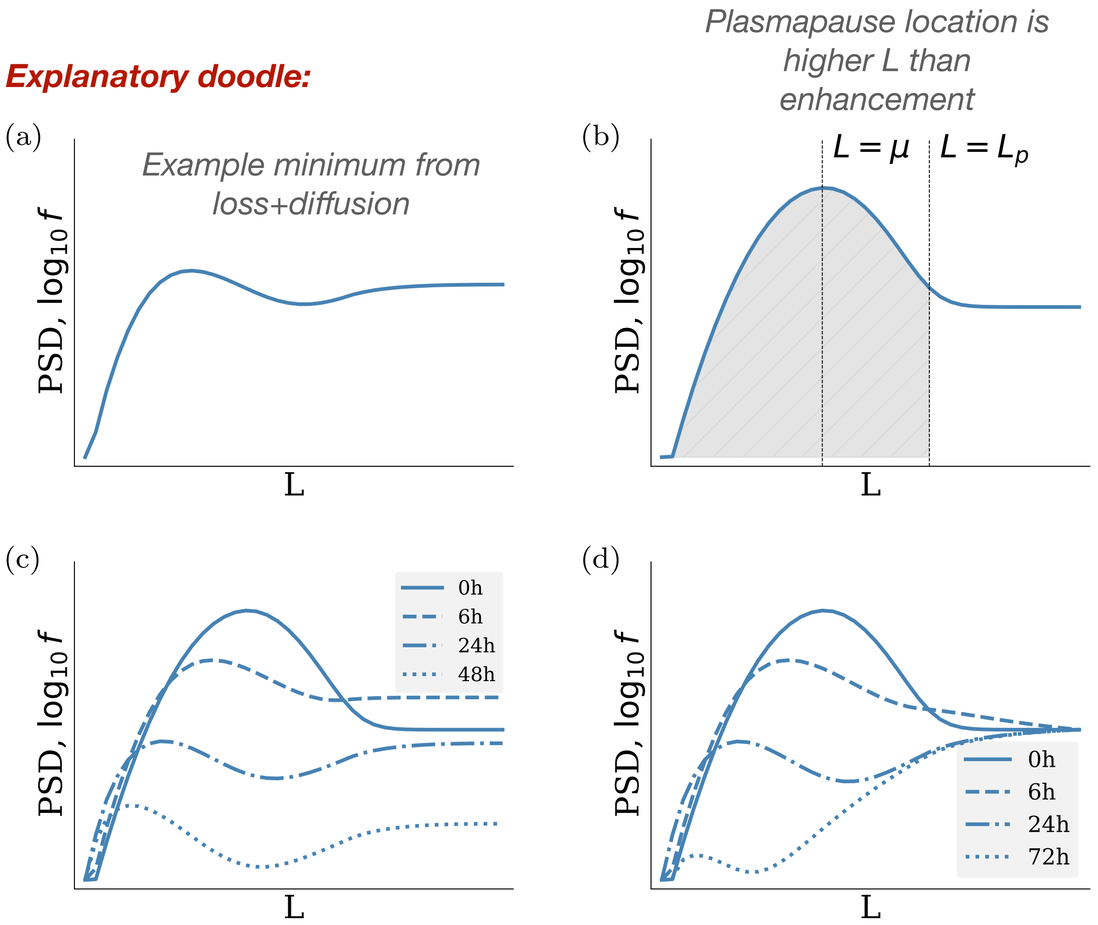
The enhancement diffuses differently depending on the outer boundary; which determines whether the enhancement can diffuse into high L.

Key results #1: t_m tells us…

Plasmapause location vs μ is important

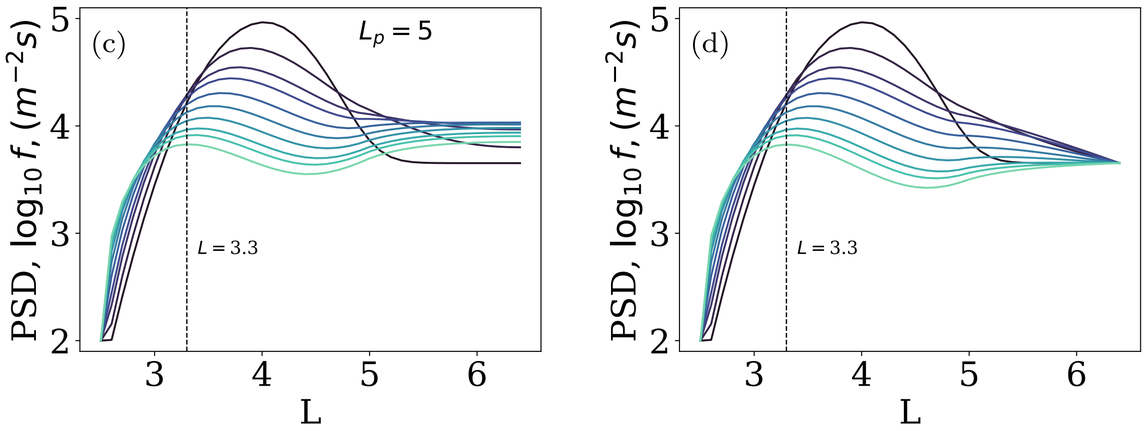
- Effects of outer boundary condition and location are dominated by loss - these are secondary, phew!
- The location of the plasmapause compared to the enhancement can result in monotonicity never being reached: t_m would need to be adapted in this case.

Explanatory doodle:



Regardless of OB condition, the minimum persists. PSD is never monotonic.

Simulations:



Key results #3: $\mathscr{E}, \mathscr{N}$

We can quantify how f moves towards equilibrium; how it diffuses

$\mathcal{N}, \mathcal{E}$ (Mass- and energy-like density)

$$\mathcal{N} = \int f \frac{1}{L^2} dL$$

Mass in experiment

$$\mathcal{E} = \frac{1}{2c_{\mathcal{E}}} \int |f|^2 \frac{1}{L^2} dL$$

How close the distribution is to a zero state; or a state that does not change with time

Using $D_{LL} = D_0 L^n$ we can exactly specify how the boundary conditions and PSD contribute to changes in $\mathcal{N}, \mathcal{E}$ over time, e.g.

$$\mathcal{E}_t = \left[\frac{D_{LL}}{L^2} f f_L \right]_{IB}^{OB} - \int \frac{D_{LL}}{L^2} (f_L)^2 \, dL$$

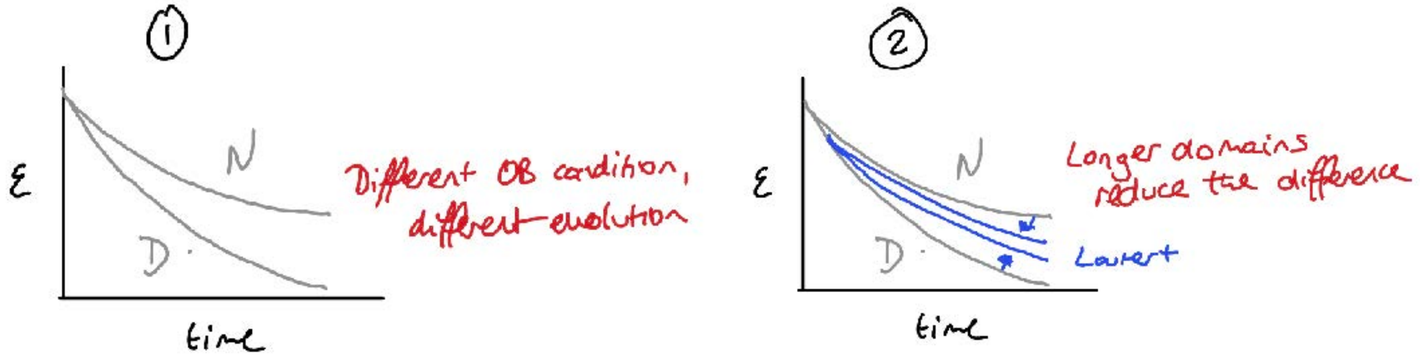
How *f* and DLL affect the rate the PSD trends toward a minimum energy state.

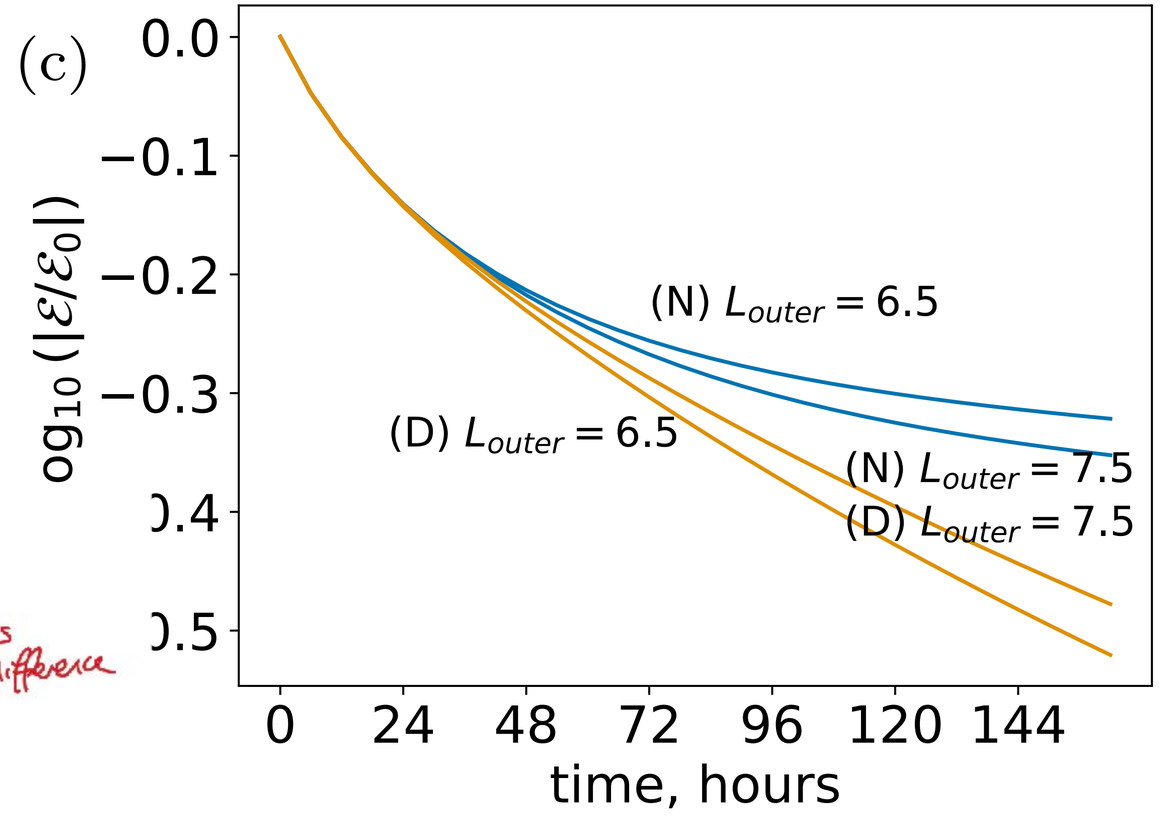
Example uses: Track the changing mass, track what conditions contribute the most to the changing PSD *f*

Key results #3: $\mathscr{E}, \mathscr{N}$

$\mathscr{E}$ over a week: outer boundary condition and location matters

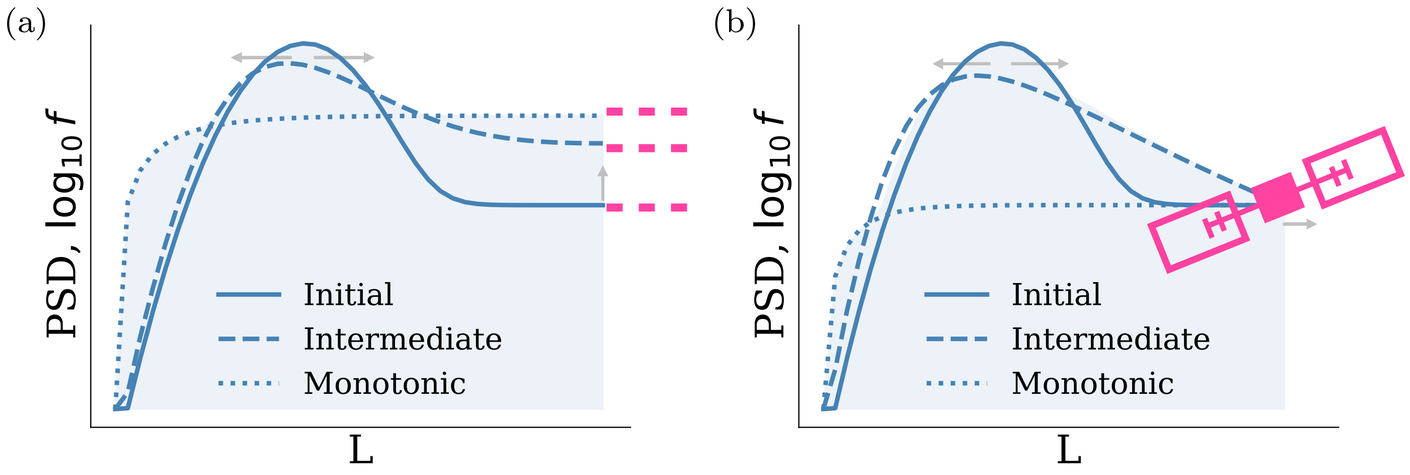
- $\mathscr{E}$ over a week
- Remember: the lower the $\mathscr{E}$, the closer we are to to low "energy"/ equilibrium state

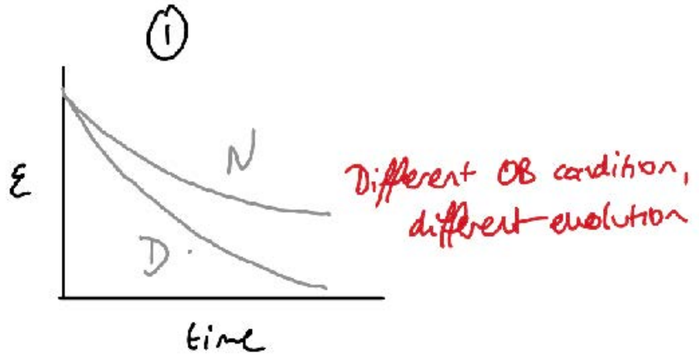




Four experiments: $\mathscr{E}$ over a week for two different outer boundary conditions and two different outer boundary locations.

Bentley et al. 2024, Physics of Plasmas







Earlier: both boundary conditions could impose unphysical processes.

Now: the length of the domain ***also*** changes the evolution.

- Cutting short a domain to where there is data will remove some of the physics.
- Lee+ 2024 showed that a shorter domain with data is better.
- This leaves big questions for forecasting, when data is unavailable.

Really, we need outer boundary **location**, **value** (data!) and different (mixed?) boundary **conditions**.

Bentley et al. 2024, Physics of Plasmas

Key result #4: Gradients > D_{LL}

$\mathscr{E}_t$ allows us to test how D_{LL} vs f_L contribute to evolution of PSD f

- Typical diffusion "acts to reduce gradients" as $\mathscr{E}_t = -\int (f_x)^2 dx$

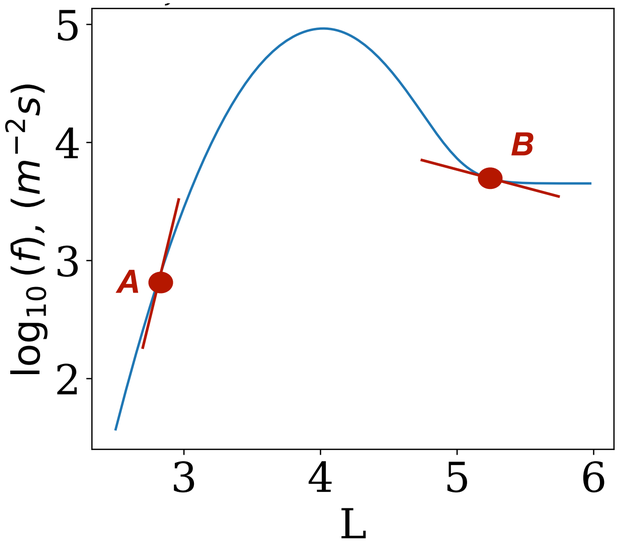
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$$\mathscr{E}_t = \left[\frac{D_{LL}}{L^2} f f_L\right]_{IB}^{OB} - \int \frac{D_{LL}}{L^2}(f_L)^2 dL$$

- Radial diffusion is more complicated!

Typical, constant-D diffusion:

Steeper gradient at A therefore more diffusion than at B



Radial diffusion:

What about D_{LL}?

Key result #4: Gradients > D_{LL}

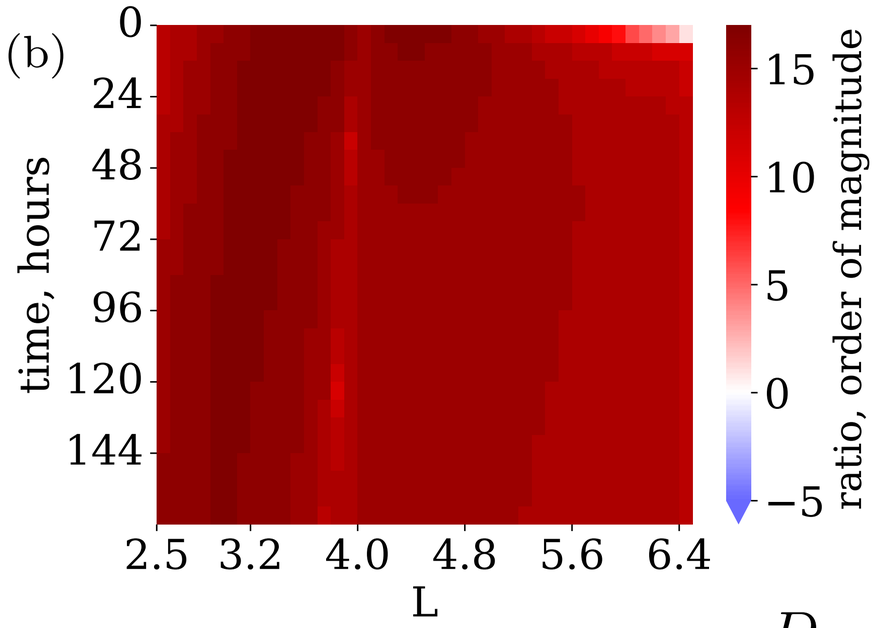
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- Radial diffusion is more complicated!

But which of D_{LL}, f_L dominate the system evolution?



Red means: $(f_L)^2 > \frac{D_{LL}}{L^2}$

Bentley et al. 2024, Physics of Plasmas

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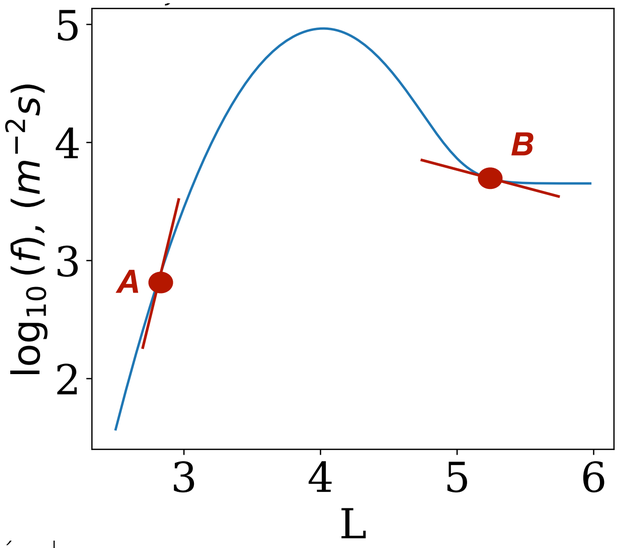
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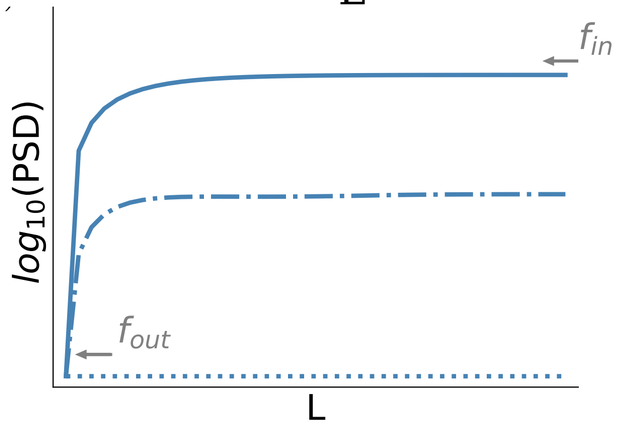
Steeper gradient at A therefore more diffusion than at B



Radial diffusion:

The same!

We end up with monotonic states that are extremely skewed to low L



Three example final, monotonic distributions.

Bentley et al. 2024, Physics of Plasmas

Conclusions

- (**KR 4**) Diffusion acts more to reduce gradients than through D_{LL}.
  - Should we focus more on determining f than D_{LL}?
  - Is this just an artefact of the 1D approximation?
- (**KR 2**) Our current handling of the outer boundary is unphysical:
  - We need the correct domain length (L_{outer}) **and** suitable numerical condition.
- (**KR 3**) Mass- and energy- like density $\mathscr{N}$, $\mathscr{E}$ are useful tools.
- (**KR 1**) Considering the morphology of the phase space density distribution helps us analyse and compare long term behaviour (e.g. via t_m)
  - Plasmapause location vs centre of enhancement: peak may never diffuse away

Bentley et al. 2024,
Physics of Plasmas;
doi:10.1063/5.0217268